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Supersymmetric SU(5) GUT with Stabilized Moduli

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Abstract

We construct a minimal example of a supersymmetric grand unified model in a toroidal compactification of type I string theory with magnetized $D9$-branes. All geometric moduli are stabilized in terms of the background internal magnetic fluxes which are of "oblique" type (mutually non-commuting). The gauge symmetry is just $SU(5)$ and the gauge non-singlet chiral spectrum contains only three families of quarks and leptons transforming in the $\mathbf{10} + \bar{\mathbf{5}}$ representations.

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1 Introduction

Closed string moduli stabilization has been intensively studied during the last years for its implication towards a comprehensive understanding of the superstring vacua [1, 2], as well as due to its significance in deriving definite low energy predictions for particle models derived from string theory. Such stabilizations employ various supergravity [1, 3], non-perturbative [2] and string theory [4–6] techniques to generate potentials for the moduli fields. However, very few examples are known so far of a complete stabilization of closed string moduli in any specific model, while the known ones are too constrained to accommodate interesting models from physical point of view. Hence, there have been very few attempts to construct a concrete model of particle physics even with partially stabilized moduli. Nevertheless, in view of the importance of the task at hand, we revisit the type I string constructions [7, 8] with moduli stabilizations [4–6], to explore the possibility of incorporating particle physics models, such as the Standard Model or GUT models based on grand unified groups, in such a framework.

A new calculable method of moduli stabilization was recently proposed, using constant internal magnetic fields in four-dimensional (4d) type I string compactifications [4, 5]. In the generic Calabi-Yau case, this method can stabilize mainly Kähler moduli [4, 9] and is thus complementary to 3-form closed string fluxes that stabilize the complex structure and the dilaton [3]. On the other hand, it can also be used in simple toroidal compactifications, stabilizing all geometric moduli in a supersymmetric vacuum using only magnetized $D9$-branes that have an exact perturbative string description [10, 11]. Ramond-Ramond (RR) tadpole cancellation requires then some charged scalar fields from the branes to acquire non-vanishing vacuum expectation values (VEVs), breaking partly the gauge symmetry in order to preserve supersymmetry [5]. Alternatively, one can break supersymmetry by D-terms and fix the dilaton at weak string coupling, by going "slightly" off-criticality and thus generating a tree-level bulk dilaton potential [12].

There are two main ingredients for this approach of moduli stabilization [4, 5]: (1) A set of nine magnetized $D9$-branes is needed to stabilize all 36 moduli of the torus T^6 by the supersymmetry conditions [13, 14]. Moreover, at least six of them must have

oblique fluxes given by mutually non-commuting matrices, in order to fix all off-diagonal components of the metric. On the other hand, all nine $U(1)$ brane factors become massive by absorbing the RR partners of the Kähler class moduli [14]. (2) Some extra branes are needed to satisfy the RR tadpole cancellation conditions, with non-trivial charged scalar VEVs turned on in order to maintain supersymmetry.

In this work, we apply the above method to construct phenomenologically interesting models. In the minimal case, three stacks of branes are needed to embed locally the Standard Model (SM) gauge group and the quantum numbers of quarks and leptons in their intersections [15]. They give rise to the gauge group $U(3) \times U(2) \times U(1)$, with the hypercharge being a linear combination of the three $U(1)$'s. Three different models can then be obtained, one of which corresponds to an $SU(5)$ Grand Unified Theory (GUT) when $U(3)$ and $U(2)$ are coincident. Here, we focus precisely on this $U(5) \times U(1)$ model employing two magnetized $D9$-brane stacks. Open strings stretched in the intersection of $U(5)$ with its orientifold image give rise to 3 chiral generations in the antisymmetric representation $\mathbf{10}$ of $SU(5)$, while the intersection of $U(5)$ with the orientifold image of $U(1)$ gives 3 chiral states transforming as $\bar{\mathbf{5}}$. Finally, the intersection of $U(5)$ with the $U(1)$ is non chiral, giving rise to Higgs pairs $\mathbf{5} + \bar{\mathbf{5}}$.

In order to obtain an odd number (3) of fermion generations, a NS-NS (Neveu-Schwarz) 2-form B-field background [16] must be turned on [17]. This requires the generalization of the minimal set of branes with oblique magnetic fluxes that generate only diagonal 5-brane tadpoles on the three orthogonal tori of $T^6 = \prod_{i=1}^{3} T_i^2$. We find indeed a set of eight such "oblique" branes which combined with $U(5)$ can fix all geometric moduli by the supersymmetry conditions. The metric is fixed in a diagonal form, depending on six radii given in terms of the magnetic fluxes. At the same time, all nine corresponding $U(1)$'s become massive yielding an $SU(5) \times U(1)$ gauge symmetry. This $U(1)$ factor cannot be made supersymmetric without the presence of charged scalar VEVs. Moreover, two extra branes are needed for RR tadpole cancellation, which also require non-vanishing VEVs to be made supersymmetric. As a result, all extra $U(1)$'s are broken and the only leftover gauge symmetry is an $SU(5)$ GUT. Furthermore, the intersections of the $U(5)$ stack with any additional brane used for moduli stabilization are non-chiral, yielding the three families

of quarks and leptons in the $\mathbf{10}+\bar{\mathbf{5}}$ representations as the only chiral spectrum of the model (gauge non-singlet).

To elaborate further, the model is described by twelve stacks of branes, namely U_5, U_1, $O_1 \ldots, O_8$, A, and B. The $SU(5)$ gauge group arises from the open string states of stack-U_5 containing five magnetized branes. The remaining eleven stacks contain only a single magnetized brane. Also, the stack-U_5 containing the GUT gauge sector, contributes to the GUT particle spectrum through open string states which either start and end on itself[1] or on the stack-U_1, having only a single brane and therefore contributing an extra $U(1)$. For this reason we will also refer to these stacks as U_5 and U_1 stacks.

The matter sector of the $SU(5)$ GUT is specified by 3 generations of fermions in the group representations $\bar{\mathbf{5}}$ and $\mathbf{10}$ of $SU(5)$, both of left-handed helicity. In the magnetized branes construction, the $\mathbf{10}$ dimensional (antisymmetric) representation of left-handed fermions:

$$\mathbf{10} \equiv \begin{pmatrix} 0 & u_3^c & u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}_{\mathbf{L}} \tag{1.1}$$

arises from the doubly charged open string states starting on the stack-U_5 and ending at its orientifold image: U_5^* and vice verse. They transform as $\mathbf{10}_{(\mathbf{2},\mathbf{0})}$ of $SU(5)\times U(1)\times U(1)$, where the first $U(1)$ refers to stack-U_5 and the second one to stack-U_1, while the subscript denotes the corresponding $U(1)$ charges. The $\bar{\mathbf{5}}$ of $SU(5)$ containing left-handed chiral fermions, or alternatively the $\mathbf{5}$ with right-handed fermions:

$$\mathbf{5} \equiv \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ \nu^c \end{pmatrix}_{\mathbf{R}} \tag{1.2}$$

[1]For simplicity, we do not distinguish a brane stack with its orientifold image, unless is explicitly stated.

are identified as states of open strings starting from stack-U_5 (with five magnetized branes) and ending on stack-U_1^* (i.e. the orientifold image of stack-U_1) and vice verse. The magnetic fluxes along the various branes are constrained by the fact that the chiral fermion spectrum, mentioned above, of the $SU(5)$ GUT should arise from these two sectors only.

Our aim, in this paper, is to give a supersymmetric construction which incorporates the above features of $SU(5)$ GUT while stabilizing all the Kähler and complex structure moduli. More precisely, for fluxes to be supersymmetric, one demands that their holomorphic $(2,0)$ part vanishes. This condition then leads to complex structure moduli stabilization [4]. In our case we show that, for the fluxes we turn on, the complex structure τ of T^6 is fixed to

$$\tau = i\, \mathbb{1}_3, \tag{1.3}$$

with $\mathbb{1}_3$ being the 3×3 identity matrix.

In this paper, we make use of the conventions given in Appendix A of Ref. [5], for the parametrization of the torus T^6, as well as for the general definitions of the Kähler and complex structure moduli. In particular, the coordinates of three factorized tori: $(T^2)^3 \in T^6$ are given by x_i, y_i $i = 1, 2, 3$ with a volume normalization:

$$\int dx_1 \wedge dy_1 \wedge dx_2 \wedge dy_2 \wedge dx_3 \wedge dy_3 = 1. \tag{1.4}$$

For Kähler moduli stabilization, we make use of the mechanism based on the magnetized D-branes supersymmetry conditions as discussed in [4, 5, 13]. Physically this corresponds to the requirement of vanishing of the potential which is generated for the moduli fields from the Fayet-Iliopoulos (FI) D-terms associated with the various branes. Even in this simplified scenario, the mammothness of the exercise is realized by noting that every magnetic flux that is introduced along any brane also induces charges corresponding to lower dimensional branes, giving rise to new tadpoles that need to be canceled. In particular, for the type I string that we are discussing, there are induced $D5$ tadpoles from fluxes along the magnetized $D9$ branes. These fluxes, in turn, are forced to be non-zero not only in order to satisfy the condition of zero net chirality among the U_5 and the extra brane stacks (except with the U_1), but in order to implement the mechanism of complex structure and Kähler moduli stabilization, as well. Specifically, for stabilizing the non-diagonal

components of the metric, one is forced to introduce 'oblique' fluxes along the $D9$-branes, thus generating 'oblique' $D5$-brane tadpoles, and all these need to be canceled.

However, as mentioned earlier, we are able to find eight brane stacks $O_1, \ldots, O_8$, with different oblique fluxes, such that the combined net induced $D5$-brane charge lies only along the three diagonal directions $[x_i, y_i]$. The holomorphicity conditions of fluxes, namely the vanishing of field strengths with purely holomorphic indices, for these brane stacks stabilizes the complex structure moduli to the value (1.3). These fluxes also introduce D-term potential for the Kähler moduli. Once the complex structure is fixed as in (1.3), the fluxes in the nine stacks $U_5, O_1, \ldots, O_8$ generate potential in such a a way that all the nine Kähler moduli, $J_{i\bar{j}}$, $(i, j = 1, 2, 3)$ are completely fixed by the D-flatness conditions, imposing the vanishing of the FI terms. The residual diagonal tadpoles of the branes in the stacks U_5, U_1, $O_1, \ldots, O_8$ are then canceled by introducing the last two brane stacks A and B. D-flatness conditions for the brane stacks U_1, A and B are also satisfied, provided some VEVs of charged scalars living on these branes are turned on to cancel the corresponding FI parameters. Magnetized D-branes provide exact CFT (conformal field theory) construction of the GUT model. However, in the presence of the these non-vanishing scalar VEVs, exact CFT description is lost. The validity of the approximation then requires these VEVs to be smaller than unity in string units, a condition which is met in our case. We explicitly determine the charged scalar VEVs and verify that they all take values $v^a << 1$. Our model therefore corresponds to the Higgsing of a magnetized $D9$-brane model to be made supersymmetric through the VEVs of certain charged scalar fields on the intersections of the branes U_1, A and B.

At this point we would like to point out that, our strategy in this paper is to start with a suitable ansatz for both the complex structure (1.3) and Kähler moduli leading to diagonal internal metric. Using this ansatz, we then determine fluxes along the branes satisfying all the constraints we elaborated upon earlier. We then use the flux solutions, to show explicitly that the moduli are indeed completely fixed, consistent with our ansatz.

The rest of the paper is organized as follows. In Section 2, we give the necessary constraints needed for building the model. This includes the discussion on moduli stabilization in subsection 2.2, the tadpole constraints in subsection 2.3 and the fermion

degeneracies in subsection 2.4. Since a crucial step in a three generation model building is the introduction of a NS-NS (Neveu-Schwarz) B-field background without which only even generation models can be built, the effect of non-zero B on the chirality and tadpoles is summarized in subsection 2.5. In Section 3, we obtain general solutions for fluxes along magnetized $D9$-branes satisfying the constraints of Section 2. Moduli stabilization is discussed in Section 4. In Section 5, the VEVs of charged scalars on the stacks U_1, A and B are determined. Our conclusions are presented in Section 6. In Appendix A, the fluxes along branes are written explicitly for the stacks $O_1, \ldots, O_8$ and the associated $D5$-brane tadpoles are given. The absence of chiral fermions is also shown from these sectors. In Appendix B, complex structure stabilization is shown explicitly using the fluxes given in Appendix A. Finally, the Kähler moduli stabilization is shown in Appendix C (as well as in Section 4).

2 Preliminaries

We now briefly review the string construction using magnetized branes, and in particular the chiral spectrum that follows for such stacks of branes due to the presence of magnetic fluxes.

2.1 Fluxes and windings

We first briefly describe the construction based on D-branes with magnetic fluxes in type I string theory, or equivalently type IIB with orientifold $O9$-planes and magnetized $D9$-branes, in a T^6 compactification. Later on, in subsection 2.5, we study the introduction of constant NS-NS B-field background in this setup.

The stacks of $D9$-branes are characterized by three independent sets of data: (a) their multiplicities N_a, (b) winding matrices $W_I^{\hat{I},a}$ and (c) 1st Chern numbers $m^a_{\hat{I}\hat{J}}$ of the $U(1)$ background on their world-volume Σ^a, $a = 1, \ldots, K$. In our case, as already stated earlier, we have $K = 12$ stacks. Also, $I, \hat{I}$ run over the target space and world-volume indices, respectively. These parameters are described below:

(a) Multiplicities: The first quantity N_a describes the rank of the the unitary gauge

group $U(N_a)$ on each $D9$ stack.

(b) Winding Matrices: The second set of parameters $W_I^{\hat{I},\,a}$ is the covering of the world-volume of each stack of $D9$-branes on the ambient space. They are characterized by the wrapping numbers of the branes around the different 1-cycles of the torus, which are encoded in the covering matrices $W_I^{\hat{I},\,a}$ defined as

$$W_J^{\hat{I}} = \frac{\partial \xi^{\hat{I}}}{\partial X^J} \qquad \text{for } \hat{I}, J = 0, \ldots, 9\,, \tag{2.1}$$

where the coordinates on the world-volume are denoted by $\xi^{\hat{I}}$, while the coordinates of the space-time $\mathcal{M}_{10}$ are X^I. Since space-time is assumed to be a direct product of a four-dimensional Minkowski manifold with a six-dimensional torus, the covering matrix is of the form:

$$W_J^{\hat{I},\,a} = \begin{pmatrix} \delta_\mu^{\hat{\mu}} & 0 \\ 0 & W_\alpha^{\hat{\alpha}\,,a} \end{pmatrix} \qquad \text{for } \mu, \hat{\mu} = 0, \ldots, 3 \text{ and } \alpha, \hat{\alpha} = 1, \ldots, 6\,, \tag{2.2}$$

with the upper block corresponding to the covering of Σ_4^a on the four-dimensional space-time $\mathcal{M}_4$. Since these are assumed to be identical, the associated covering map $W_\mu^{\hat{\mu}}$ is the identity, $W_\mu^{\hat{\mu}} = \delta_\mu^{\hat{\mu}}$. The entries of the lower block, on the other hand, describe the wrapping numbers of the $D9$-branes around the different 1-cycles of the torus T^6 which are therefore restricted to be integers $W_\alpha^{\hat{\alpha}} \in \mathbb{Z}$, $\forall\, \alpha, \hat{\alpha} = 1, \ldots, 6$ [6].

For simplicity, in the examples we consider here, the winding matrix $W_\alpha^{\hat{\alpha}}$ in the internal directions is also chosen to be a six-dimensional diagonal matrix, implying an embedding such that the six compact $D9$ world-volume coordinates are identified with those of the internal target space T^6, up to a winding multiplicity factor n_α^a, for a brane stack-a:

$$n_\alpha^a \equiv W_\alpha^{\hat{\alpha},a}. \tag{2.3}$$

We will also use the notation

$$\hat{n}_1^a \equiv n_1^a n_2^a, \quad \hat{n}_2^a \equiv n_3^a n_4^a, \quad \hat{n}_3^a \equiv n_5^a n_6^a, \quad \text{(no sum on a)} \tag{2.4}$$

to define the diagonal wrapping of the $D9$'s on the three orthogonal T^2's inside T^6, given by:

$$x^i \equiv X^\alpha, \;\; \alpha = 1, 3, 5\,; \quad y^i \equiv X^\alpha, \;\; \alpha = 2, 4, 6\,, \tag{2.5}$$

with periodicities: $x^i = x^i + 1$, $y^i \equiv y^i + 1$:

$$\mathbb{T}^6 = \bigotimes_{i=1}^{3} \mathbb{T}_i^2 \,, \tag{2.6}$$

and coordinates of the orthogonal 2-tori (T_i^2) being (x^i, y^i) for $i = 1, 2, 3$.

For further simplification, in our example, we will choose for all stacks trivial windings:

$$n_\alpha^a \equiv W_\alpha^{\hat{\alpha},a} = 1, \text{ for } \alpha = 1, .., 6, \quad a = U_5,\, U_1,\, O_1 \cdots O_8,\, A,\, B. \tag{2.7}$$

However in this section, in order to describe the formalism, we keep still general winding matrices $W_\alpha^{\hat{\alpha},a}$.

(c) First Chern numbers: The parameters $m^a_{\hat{I}\hat{J}}$ of the brane data given above are the 1st Chern numbers of the $U(1) \subset U(N_a)$ background on the world-volume of the $D9$-branes. For each stack $U(N_a) = U(1)_a \times SU(N_a)$, the $U(1)_a$ has a constant field strength on the covering of the internal space. These are subject to the Dirac quantization condition which implies that all internal magnetic fluxes $F^a_{\hat{\alpha}\hat{\beta}}$, on the world-volume of each stack of $D9$-branes, are integrally quantized.

Explicitly, the world-volume fluxes $F^a_{\hat{\alpha}\hat{\beta}}$ and the corresponding target space induced fluxes $p^a_{\alpha\beta}$ are quantized as

$$\begin{cases} F^a_{\hat{\alpha}\hat{\beta}} = m^a_{\hat{\alpha}\hat{\beta}} \in \mathbb{Z} & \forall\, \hat{\alpha}, \hat{\beta} = 1, \ldots, 6 \\ p^a_{\alpha\beta} = (W^{-1})_\alpha^{\hat{\alpha},\, a} (W^{-1})_\beta^{\hat{\beta},\, a}\, m^a_{\hat{\alpha}\hat{\beta}} \in \mathbb{Q}, & \forall\, \alpha, \beta = 1, \ldots, 6 \end{cases} \quad \forall a = 1, \ldots, K\,. \tag{2.8}$$

For later use, when fluxes are turned on only along three factorized T^2's of eq. (2.6), as will be the case for some of our brane stacks, we make use of the following convenient notation:

$$\hat{m}_1^a \equiv m_{12}^a \equiv m^a_{x^1y^1}, \quad \hat{m}_2^a \equiv m_{34}^a \equiv m^a_{x^2y^2}, \quad \hat{m}_3^a \equiv m_{56}^a \equiv m^a_{x^3y^3}. \tag{2.9}$$

The magnetized $D9$-branes couple only to the $U(1)$ flux associated with the gauge fields located on their own world-volume. In other words, the charges of the endpoints q_R and q_L of the open strings stretched between the i-th and the j-th $D9$-brane can be written as $q_L \equiv q_i$ and $q_R \equiv -q_j$, while the Cartan generator h is given by $h = \mathrm{diag}(h_1\, 1\!\!1_{N_1}, \ldots, h_N\, 1\!\!1_{N_K})$, with $1\!\!1_{N_a}$ being the $N_a \times N_a$ identity matrix. In addition, in

type I string theory, the number of magnetized $D9$-branes must be doubled. Since the orientifold projection $\mathcal{O} = \Omega_p$ is defined by the world-sheet parity, it maps the field strength $F_a = dA_a$ of the $U(1)_a$ gauge potential A_a to its opposite, $\mathcal{O} : F_a \to -F_a$. Therefore, the magnetized $D9$-branes are not an invariant configuration and for each stack a mirror stack must be added with opposite flux on its world-volume.

2.2 Stabilization

We now write down the supersymmetry conditions for magnetized $D9$-branes in the context of type I toroidal compactifications and discuss the stabilization of complex structure and Kähler class moduli using such conditions.

The geometric moduli of T^6 decompose in a complex structure variation which is parametrized by the matrix τ_{ij} entering in the definition of the complex coordinates

$$z_i = x_i + \tau_{ij} y^j \,, \tag{2.10}$$

and in the Kähler variation of the mixed part of the metric described by the real $(1,1)$-form $J = i\delta g_{i\bar{j}} dz^i \wedge d\bar{z}^j$. The supersymmetry conditions then read [4,5]:

$$F^a_{(2,0)} = 0 \;\; ; \quad \mathcal{F}_a \wedge \mathcal{F}_a \wedge \mathcal{F}_a = \mathcal{F}_a \wedge J \wedge J \;\; ; \quad \det W_a \left(J \wedge J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a \wedge J \right) > 0 \,, \tag{2.11}$$

for each $a = 1, \ldots, K$. The complexified fluxes can be written as

$$F^a_{(2,0)} \;=\; (\tau - \bar{\tau})^{-1^T} \left[\tau^T p^a_{xx} \tau - \tau^T p^a_{xy} - p^a_{yx} \tau + p^a_{yy} \right] (\tau - \bar{\tau})^{-1}, \tag{2.12}$$

$$F^a_{(1,1)} \;=\; (\tau - \bar{\tau})^{-1^T} \left[-\tau^T p^a_{xx} \bar{\tau} + \tau^T p^a_{xy} + p^a_{yx} \bar{\tau} - p^a_{yy} \right] (\tau - \bar{\tau})^{-1}, \tag{2.13}$$

where the matrices $(p^a_{x^i x^j})$, $(p^a_{x^i y^j})$ and $(p^a_{y^i y^j})$ are the quantized field strengths in target space, given in eq. (2.8). For our choice (2.7), they coincide with the Chern numbers m^a along the corresponding cycles. The field strengths $F^a_{(2,0)}$ and $F^a_{(1,1)}$ are 3×3 matrices that correspond to the upper half of the matrix $\mathcal{F}^a$:

$$\mathcal{F}^a \equiv -(2\pi)^2 i\alpha' \begin{pmatrix} F^a_{(2,0)} & F^a_{(1,1)} \\ -{F^a}^\dagger_{(1,1)} & {F^{a*}}_{(2,0)} \end{pmatrix} \,, \tag{2.14}$$

which is the total field strength in the cohomology basis $e_{i\bar{j}} = i dz^i \wedge d\bar{z}^j$ [4,5].

The first set of conditions of eq. (2.11) states that the purely holomorphic flux vanishes. For given flux quanta and winding numbers, this matrix equation restricts the complex structure τ. Using eq. (2.12), the supersymmetry conditions for each stack can first be seen as a restriction on the parameters of the complex structure matrix elements τ:

$$F^a_{(2,0)} = 0 \qquad \rightarrow \qquad \tau^T p^a_{xx} \tau - \tau^T p^a_{xy} - p^a_{yx} \tau + p^a_{yy} = 0\,, \tag{2.15}$$

giving rise to at most six complex equations for each brane stack a.

The second set of conditions of eq. (2.11) gives rise to a real equation and restricts the Kähler moduli. This can be understood as a D-flatness condition. In the four-dimensional effective action, the magnetic fluxes give rise to topological couplings for the different axions of the compactified field theory. These arise from the dimensional reduction of the Wess Zumino action. In addition to the topological coupling, the $\mathcal{N} = 1$ supersymmetric action yields a Fayet-Iliopoulos (FI) term of the form:

$$\frac{\xi_a}{g_a^2} = \frac{1}{(4\pi^2\alpha')^3} \int_{T^6} \big(\mathcal{F}_a \wedge \mathcal{F}_a \wedge \mathcal{F}_a - \mathcal{F}_a \wedge J \wedge J\big)\,. \tag{2.16}$$

The D-flatness condition in the absence of charged scalars requires then that $<\mathrm{D}_a>= \xi_a = 0$, which is equivalent to the second equation of eq. (2.11). Finally, the last inequality in eq. (2.11) may also be understood from a four-dimensional viewpoint as the positivity of the $U(1)_a$ gauge coupling g_a^2, since its expression in terms of the fluxes and moduli reads

$$\frac{1}{g_a^2} = \frac{1}{(4\pi^2\alpha')^3} \int_{T^6} \big(J \wedge J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a \wedge J\big)\,. \tag{2.17}$$

The above supersymmetry conditions, get modified in the presence of VEVs for scalars charged under the $U(1)$ gauge groups of the branes. The D-flatness condition, in the low energy field theory approximation, then reads:

$$\mathrm{D}_a = -\left(\sum_\phi q_a^\phi |\phi|^2 G_\phi + M_s^2 \xi_a\right) = 0\,, \tag{2.18}$$

where $M_s = \alpha'^{-1/2}$ is the string scale[2], and the sum is extended over all scalars ϕ charged under the a-th $U(1)_a$ with charge q_a^ϕ and metric G_ϕ. Such scalars arise in the compactification of magnetized $D9$-branes in type I string theory, for instance from the NS sector

[2]When mass scales are absent, string units are implicit throughout the paper.

of open strings stretched between the a-th brane and its image $a^\star$, or between the stack-a and another stack-b or its image b^*. When one of these scalars acquire a non-vanishing VEV $\langle|\phi|\rangle^2 = v_\phi^2$, the calibration condition of eq. (2.11) is modified to:

$$q_a v_a^2 \int_{T^6} \big(J\wedge J\wedge J - \mathcal{F}_a\wedge\mathcal{F}_a\wedge J\big) \; = \; -\frac{M_s^2}{G}\int_{T^6}\big(\mathcal{F}_a\wedge\mathcal{F}_a\wedge\mathcal{F}_a - \mathcal{F}_a\wedge J\wedge J\big) \tag{2.19}$$

$$\det W_a \left(J\wedge J\wedge J - \mathcal{F}_a\wedge\mathcal{F}_a\wedge J\right) \; > \; 0 \quad , \quad \forall a = 1,\ldots,K\,. \tag{2.20}$$

Note that our computation is valid for small values of v_a (in string units), since the inclusion of the charged scalars in the D-term is in principle valid perturbatively.

Actually, the fields appearing in (2.18) are not canonically normalized since the metric G_ϕ appears explicitly also in their kinetic terms. Thus, the physical VEV is $v_\phi\sqrt{G_\phi}$. However, to estimate the validity of the perturbative approach, it is more appropriate to keep v_ϕ instead of $v_\phi\sqrt{G_\phi}$. The reason is that the next to leading correction to the D-term involves a quartic term of the type $|\phi|^4$, proportional to a new coefficient $\mathcal{K}$, and the condition of validity of perturbation theory is $\mathcal{K}v_\phi^2/G_\phi << 1$. A rough estimate is then obtained by approximating $\mathcal{K}\sim G_\phi$, which gives our condition.

The metric G_ϕ of the scalars living on the brane has been computed explicitly for the case of diagonal fluxes [18]. In this special case, the fluxes are denoted by three angles θ_i^a, $(i = 1, 2, 3)$.[3] Then suppressing index-a, we have:

$$\tan\pi\theta_i = \frac{p_{x^iy^i}}{J_i} \equiv \frac{(F_{(1,1)})_{z^i\bar{z}^i}}{J_i}, \tag{2.21}$$

and

$$G = e^{\gamma_E(\theta_1+\theta_2+\theta_3)} \times \sqrt{\frac{\Gamma(\theta_1)\Gamma(\theta_2)\Gamma(\theta_3)}{\Gamma(1-\theta_1)\Gamma(1-\theta_2)\Gamma(1-\theta_3)}}, \tag{2.22}$$

with γ_E being the Euler constant. The above results will be applied in section 5 to find out the FI parameters and charged scalar VEVs along three of the twelve brane stacks: U_1, A and B. The other nine stacks, U_5, $O_1,\ldots,O_8$, stabilizing all the geometric moduli, will satisfy the calibration condition $\xi^a = 0$ in the absence of open string scalar VEVs. Moreover, the RR moduli that appear in the same chiral multiplets as the geometric Kähler moduli, become Goldstone modes which get absorbed by the $U(1)$ gauge bosons [4] corresponding to each of the D-terms that stabilize the relevant geometric moduli.

[3] See examples in Appendix A for the precise map between $p_{x^iy^i}$ and $(F_{(1,1)})_{z^i\bar{z}^i}$.

2.3 Tadpoles

In toroidal compactifications of type I string theory, the magnetized $D9$-branes induce 5-brane charges as well, while the 3-brane and 7-brane charges automatically vanish due to the presence of mirror branes with opposite flux. For general magnetic fluxes, RR tadpole conditions can be written in terms of the Chern numbers and winding matrix [5, 6] as:

$$16 = \sum_{a=1}^{K} N_a \, \mathrm{det}W_a \equiv \sum_{a=1}^{K} Q^{9,\,a}, \tag{2.23}$$

$$0 = \sum_{a=1}^{K} N_a \, \mathrm{det}W_a \; \mathcal{Q}^{a,\,\alpha\beta} \equiv \sum_{a=1}^{K} Q^{5,\,a}_{\alpha\beta}, \qquad \forall \alpha, \beta = 1, \ldots, 6 \tag{2.24}$$

where

$$\mathcal{Q}^{a,\,\alpha\beta} = \epsilon^{\alpha\beta\delta\gamma\sigma\tau} p^a_{\delta\gamma} p^a_{\sigma\tau} \,.$$

The l.h.s. of eq. (2.23) arises from the contribution of the $O9$-plane. On the other hand, in toroidal compactifications there are no $O5$-planes and thus the l.h.s. of eq. (2.24) vanishes.

For our choice of windings (2.7), $W_i^{\hat{i}} = 1$, the $D9$ tadpole contribution from a given stack-a of branes is simply equal to the number of branes, N_a. The $D5$ tadpole expression also takes a simple form for the fluxes satisfying the $F^a_{(2,0)} = 0$ condition (2.11). The fluxes are then represented by three-dimensional Hermitian matrices ($F^a_{(1,1)}$) which appeared in eq. (2.14) and the $D5$ tadpoles $\mathcal{Q}^{5,\,a}_{i\bar{j}}$ are the Cofactors of the $i\bar{j}$ matrix elements $(F^a_{(1,1)})_{i\bar{j}}$. Fluxes and tadpoles in such a form are given in Appendix A.

2.4 Spectrum

The gauge sector of the spectrum follows from the open string states corresponding to strings starting and ending on the same brane stack. The gauge symmetry group is given by a product of unitary groups $\otimes_a U(N_a)$, upon identification of the associated open strings attached on a given stack with the ones attached on the mirror (under the orientifold transformation) stack. In addition to these vector bosons, the massless spectrum contains adjoint scalars and fermions forming $\mathcal{N} = 4$, $d = 4$ supermultiplets.

In the matter sector, the massless spectrum is obtained from the following open string states [14, 19]:

1. Open strings stretched between the a-th and b-th stack give rise to chiral spinors in the bifundamental representation $(N_a, \bar{N}_b)$ of $U(N_a) \times U(N_b)$. Their multiplicity I_{ab} is given by [6]:

$$I_{ab} = \frac{\det W_a \det W_b}{(2\pi)^3} \int_{T^6} \left(q_a F^a_{(1,1)} + q_b F^b_{(1,1)} \right)^3 , \tag{2.25}$$

where $F^a_{(1,1)}$ (given in eqs. (2.13) and (2.14)) is the pullback of the integrally quantized world-volume flux $m^a_{\hat{\alpha}\hat{\beta}}$ on the target torus in the complex basis (2.10), and q_a is the corresponding $U(1)_a$ charge; in our case $q_a = +1$ (-1) for the fundamental (anti-fundamental representation). The transformation under the gauge group and their multiplicities are thus determined in terms of the data $(N_a, W_I^{\hat{I},a}, m_{\hat{I}\hat{J}})$.

For factorized toroidal compactifications $(T^2)^3$ (2.6) with only diagonal fluxes $p_{x^i y^i}$ $(i = 1, 2, 3)$, the multiplicities of chiral fermions, arising from strings starting from stack a and ending at b or vice versa, take the simple form (using notations of eqs. (2.4) and (2.9)):

$$\begin{aligned} (N_a, \overline{N}_b) : I_{ab} &= \prod_i (\hat{m}^a_i \hat{n}^b_i - \hat{n}^a_i \hat{m}^b_i), \\ (N_a, N_b) : I_{ab^*} &= \prod_i (\hat{m}^a_i \hat{n}^b_i + \hat{n}^a_i \hat{m}^b_i) . \end{aligned} \tag{2.26}$$

where i is the label of the i-th two-tori T^2_i, and the integers $\hat{m}^a_i, \hat{n}^a_i$ enter in the multiplicity expressions through the magnetic field as in eq. (2.8).

In the model that we construct, however, we need stacks with fluxes which contain both diagonal and oblique flux components, for the purpose of complete Kähler and complex structure moduli stabilization.

2. Open strings stretched between the a-th brane and its mirror $a^\star$ give rise to massless modes associated to $I_{aa^\star}$ chiral fermions. These transform either in the antisymmetric or symmetric representation of $U(N_a)$. For factorized toroidal compactifications $(T^2)^3$, the multiplicities of chiral fermions are given by;

$$\begin{aligned} \text{Antisymmetric :} \quad & \frac{1}{2} \left(\prod_i 2\hat{m}^a_i \right) \left(\prod_j \hat{n}^a_j + 1 \right) , \\ \text{Symmetric :} \quad & \frac{1}{2} \left(\prod_i 2\hat{m}^a_i \right) \left(\prod_j \hat{n}^a_j - 1 \right) . \end{aligned} \tag{2.27}$$

In generic configurations, where supersymmetry is broken by the magnetic fluxes, the scalar partners of the massless chiral spinors in twisted open string sectors (*i.e.* from non-trivial brane intersections) are massive (or tachyonic). Moreover, when a chiral index I_{ab} vanishes, the corresponding intersection of stacks a and b is non-chiral. The multiplicity of the non-chiral spectrum is then determined by extracting the vanishing factor and calculating the corresponding chiral index in higher dimensions. This is done explicitly for our model below, in section 3.7.

2.5 Constant NS-NS B-field backround

In toroidal models with vanishing B-field, the net generation number of chiral fermions is in general even [17]. Thus, it is necessary to turn on a constant B-field background in order to obtain a Standard Model like spectrum with three generations. Due to the world-sheet parity projection Ω, the NS-NS two-index field $B_{\alpha\beta}$ is projected out from the physical spectrum and constrained to take the discrete values 0 or 1/2 (in string units) along a 2-cycle $(\alpha\beta)$ of T^6 [16].

For branes at angles, $B_{\alpha\beta} = 1/2$ changes the number of intersection points of the two branes. For the case of magnetized $D9$-branes, if B is turned on only along the three diagonal 2-tori:

$$B_{x^i y^i} \equiv b_i = \frac{1}{2}\,, \quad i = 1, 2, 3, \tag{2.28}$$

the effect is accounted for by introducing an effective world-volume magnetic flux quantum, defined by $\tilde{\hat{m}}_j^a = \hat{m}_j^a + \frac{1}{2}\hat{n}_j^a$, while the first Chern numbers along all other 2-cycles remain unchanged (and integral). Thus, the modification can be summarized by

$$(\hat{m}_j^a, \hat{n}_j^a) \ \text{ for } \ b_j = 0 \quad \rightarrow (\hat{m}_j^a + \frac{1}{2}\hat{n}_j^a, \hat{n}_j^a) \equiv (\tilde{\hat{m}}_j^a, \hat{n}_j^a)\,, \ \text{ for } b_j = \frac{1}{2}, \tag{2.29}$$

along the particular 2-cycles where the NS-NS B-field is turned on. This transformation also takes into account the changes in the fermion degeneracies given in eqs. (2.26) and (2.27) (as well as in (2.33), (2.34) below), due to the presence of a non-zero B:

$$(N_a, \overline{N}_b) : I_{ab} = \prod_i (\tilde{\hat{m}}_i^a \hat{n}_i^b - \hat{n}_i^a \tilde{\hat{m}}_i^b),$$

$$(N_a, N_b) : I_{ab^*} = \prod_i (\tilde{\tilde{m}}_i^a \hat{n}_i^b + \hat{n}_i^a \tilde{\tilde{m}}_i^b)\,, \tag{2.30}$$

$$\text{Antisymmetric} : I_{aa^*}^A = \quad \frac{1}{2}\left(\prod_i 2\tilde{\tilde{m}}_i^a\right)\left(\prod_j \hat{n}_j^a + 1\right), \tag{2.31}$$

$$\text{Symmetric} : I_{aa^*}^S = \quad \frac{1}{2}\left(\prod_i 2\tilde{\tilde{m}}_i^a\right)\left(\prod_j \hat{n}_j^a - 1\right). \tag{2.32}$$

In addition, similar modifications take place in the tadpole cancellation conditions, as well. Note that for non trivial B, if $\hat{n}_i^a$ is odd $\tilde{\tilde{m}}_i^a$ is half-integer, while if $\hat{n}_i^a$ is even $\tilde{\tilde{m}}_i^a$ must be integer.

When restricting to the trivial windings of eq. (2.7) that we use in this paper, $\hat{n}_i^a = 1$, the degeneracy formula (2.25) simplifies to:

$$(N_a, \overline{N}_b) : I_{ab} = \det\left(\tilde{F}_{(1,1)}^a - \tilde{F}_{(1,1)}^b\right)\,, \tag{2.33}$$

$$(N_a, N_b) : I_{ab^*} = \det\left(\tilde{F}_{(1,1)}^a + \tilde{F}_{(1,1)}^b\right), \tag{2.34}$$

where $\tilde{F} = F + B$ and we have assumed the canonical volume normalization (1.4) on T^6. Similarly, the multiplicity of chiral antisymmetric representations is given by:

$$\text{Antisymmetric} : \quad I_{aa^*}^A = \prod_i \left(2\tilde{\tilde{m}}_i^a\right)\,, \tag{2.35}$$

while there are no states in symmetric representations. Finally, the tadpole cancellation conditions (2.23) and (2.24) become:

$$\sum_{a=1}^{K} N_a = 16 \quad ; \quad \sum_{a=1}^{K} N_a \,\mathrm{Co}(\tilde{F}_{(1,1)}^a)_{i\bar{j}} = 0 \qquad \forall\, i, j = 1, \ldots, 3\,. \tag{2.36}$$

3 Constructing a three generation $SU(5)$ GUT model

In this section, we first present in subsection 3.1 the brane stacks U_5 and U_1, on which the $SU(5)$ GUT, with three generations of chiral fermions, lives. Then, in subsection 3.2, we write down the conditions which any extra stacks, called O_a have to satisfy, so that there are no net $SU(5)$ non-singlet chiral fermions corresponding to open strings of the type: $U_5 - O_a$ and $U_5 - O_a^*$. In other words:

$$I_{U_5 O_a} + I_{U_5 O_a^*} = 0. \tag{3.1}$$

In addition, we also write down, in subsection 3.3, the condition that such stacks are mutually supersymmetric with the stack U_5, without turning on any charged scalar VEVs on these branes. The solution of these conditions giving eight branes $O_1, ..., O_8$ is presented in subsections 3.4 and 3.5. They are all supersymmetric, stabilize all Kähler moduli (together with stack-U_5) and cancel all tadpoles along the oblique directions, $x_i x_j$, $x_i y_j$, $y_i y_j$ for $i \neq j$. Finally in subsection 3.6, two more stacks A and B are found which cancel the overall $D9$ and $D5$-brane tadpoles (together with the U_1 stack).

As stated earlier, our strategy to find solutions for branes and fluxes is to first assume a canonical complex structure and Kähler moduli which have non-zero components only along the three factorized orthogonal 2-tori. In other words, we look for solutions where Kähler moduli are eventually stabilized such that

$$J_{i\bar{j}} = 0, \ \ i \neq j, \ \ (i,j = 1,2,3). \tag{3.2}$$

By assuming the complex structure and Kähler moduli as in eqs. (1.3) and (3.2), we then find fluxes needed to be turned on in order to cancel tadpoles. These fluxes are also used in the stabilization equations, in section 4 and Appendices B and C, to show that moduli are indeed completely fixed in a way that the six-torus metric becomes diagonal.

3.1 SU(5) GUT brane stacks

We now present the two brane stacks U_5 and U_1 which give the particle spectrum of $SU(5)$ GUT. For this purpose, we consider diagonally magnetized $D9$-branes on a factorized six-dimensional internal torus (2.6), in the presence of a NS-NS B-field turned on according to eq. (2.28). The stacks of $D9$-branes have multiplicities $N_{U_5} = 5$ and $N_{U_1} = 1$, so that an $SU(5)$ gauge group can be accommodated on the first one. Next, we impose a constraint on the windings $\hat{n}_i^{U_5}$ (defined in eq.(2.4)) of this stack by demanding that chiral fermion multiplicities in the symmetric representation of $SU(5)$ is zero. Then from eqs. (2.32), we obtain the constraint:

$$\prod_j \hat{n}_j^{U_5} = 1. \tag{3.3}$$

We solve eq. (3.3) by making the choice (2.7): $n_\alpha^{U_5} \equiv W_\alpha^{\hat{\alpha}, U_5} = 1$ for the stack U_5. This also implies $\hat{n}_i^{U_5} = 1$ for $i = 1,2,3$. Moreover, since from (2.23) the total $D9$-brane charge has

to be sixteen and higher winding numbers give larger contributions to the $D9$ tadpole, the windings in all stacks will be restricted[4] to $n_i^a = 1$ so that a maximum number of brane stacks can be accommodated (with $Q^9 = 16$), in view of fulfilling the task of stabilization.

Indeed, the stack U_5 already saturates five units of $D9$ charge while stabilizing only a single Kähler modulus. One more unit of $D9$ charge is saturated by the U_1 stack, responsible for producing the chiral fermions in the representation $\bar{\mathbf{5}}$ of $SU(5)$ at its intersection with U_5. Moreover, it cannot be made supersymmetric in the absence of charged scalar VEVs, as we will see below. Thus, stabilization of the eight remaining Kähler moduli, apart from the one stabilized by the U_5 stack, needs eight additional branes $O_1, \ldots, O_8$, contributing at least that many units of $D9$ charge (when windings are all one). These leave only two units of $D9$ charge yet to be saturated, which are also required to cancel any $D5$-brane tadpoles generated by the ten stacks, U_5, U_1 and $O_1, \ldots, O_8$. We find that this is achieved by two stacks A and B, also of windings one, so that the total $D9$ charge is $Q^9 = 16$ and all $D5$ tadpoles vanish $Q^5_{\alpha\beta} = 0$.

Now, after having imposed the condition that symmetric doubly charged representations of $SU(5)$ are absent, we find solutions for the first Chern numbers and fluxes, so that the the degeneracy of chiral fermions in the antisymmetric representation $\mathbf{10}$ is equal to three. These multiplicities are given in eqs. (2.31), (2.35), and when applied to the stack U_5 give the constraint:

$$(2\hat{m}_1^{U_5} + 1)(2\hat{m}_2^{U_5} + 1)(2\hat{m}_3^{U_5} + 1) = 3, \tag{3.4}$$

with a solution:

$$\hat{m}_1^{U_5} = -2, \;\; \hat{m}_2^{U_5} = -1, \;\; \hat{m}_3^{U_5} = 0. \tag{3.5}$$

The corresponding flux components are:

$$p^{U_5}_{x^1y^1} = -\frac{3}{2}, \;\; p^{U_5}_{x^2y^2} = -\frac{1}{2}, \;\; p^{U_5}_{x^3y^3} = \frac{1}{2}, \tag{3.6}$$

associated to the total (target space) flux matrix

$$\tilde{F}^{U_5}_{(1,1)} = \begin{pmatrix} -\frac{3}{2} & & \\ & -\frac{1}{2} & \\ & & \frac{1}{2} \end{pmatrix}. \tag{3.7}$$

[4]detW is restricted to be positive definite in order to avoid the presence of anti-branes.

At this level, the choice of signs is arbitrary and is taken for convenience.

Next, we solve the condition for the presence of three generations of chiral fermions transforming in $\bar{\mathbf{5}}$ of $SU(5)$. These come from singly charged open string states starting from the U_5 stack and ending on the U_1 stack or its image. In other words, we use the condition:

$$I_{U_5U_1} + I_{U_5U_1^*} = -3. \tag{3.8}$$

To solve this condition for diagonal fluxes, one can use the formulae (2.30), or alternatively eqs. (2.33) and (2.34). In the presence of the NS-NS $B_{\alpha\beta}$-field of our choice (2.28), and using the fluxes along the U_5 stack (3.6) or (3.7), the formulae take a form:

$$(N_{U_5}, \overline{N}_{U_1}): \quad I_{U_5U_1} = (-\frac{3}{2} - F_1^{U_1})(-\frac{1}{2} - F_2^{U_1})(\frac{1}{2} - F_3^{U_1})\,, \tag{3.9}$$

$$(N_{U_5}, N_{U_1}): \quad I_{U_5U_1^*} = (-\frac{3}{2} + F_{U_1}^1)(-\frac{1}{2} + F_2^{U_1})(\frac{1}{2} + F_3^{U_1})\,, \tag{3.10}$$

where we have used the notation $F_i^a \equiv (\tilde{F}^a_{(1,1)})_{i\bar{i}}$ for a given stack-a. We will also demand that all components $F_1^{U_1}, F_2^{U_1}, F_3^{U_1}$ are half-integers, due to the shift in 1st Chern numbers $\hat{m}_i^{U_1}$ by half a unit, in the presence of a non-zero NS-NS B-field along the three T^2's (2.6). We then get a solution of eq. (3.8):

$$I_{U_5U_1} = 0, \;\; I_{U_5U_1^*} = -3, \tag{3.11}$$

for flux components on the stack U_1:

$$F_1^{U_1} = -\frac{3}{2}, \;\; F_2^{U_1} = \frac{3}{2}, \;\; F_3^{U_1} = \frac{1}{2}. \tag{3.12}$$

One can ask whether solutions other than (3.12) are possible for the U_1 stack. For instance, instead of the choice $(0, -3)$ of eq. (3.11) for the intersections $U_5 - U_1$ and $U_5 - U_1^*$ subject to the condition (3.8), one could try $(-3, 0)$ or in general $(n, -n-3)$, for n any integer. Note that n (for $n > 0$) or $-n-3$ (for $n < -3$) is the number of electroweak Higgs pairs contained in $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$. Thus, the cases $(-1, -2)$ and $(-2, -1)$ were excluded because of the absence of higgses, but other cases such as $n = 1$ or $n = -4$ (containing one Higgs pair) are worth to explore. We leave these as exercises for the future.

The present results, including the quanta $(\hat{m}_i, \hat{n}_i)$ for both U_5 and U_1 stacks, are summarized in Table 1. Moreover, the (chiral) massless spectrum under the resulting gauge

Stack no. a	No. of branes: N_a	Windings $(\hat{n}_1^a, \hat{n}_2^a, \hat{n}_3^a)$	Chern no. $(\hat{m}_1^a, \hat{m}_2^a, \hat{m}_3^a)$	Fluxes $[\frac{(\hat{m}_1^a+\hat{n}_1^a/2)}{\hat{n}_1^a}, \frac{(\hat{m}_2^a+\hat{n}_2^a/2)}{\hat{n}_2^a}, \frac{(\hat{m}_3^a+\hat{n}_3^a/2)}{\hat{n}_3^a}]$
Stack-U_5	5	$(1,1,1)$	$(-2,-1,0)$	$[-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}]$
Stack-U_1	1	$(1,1,1)$	$(-2,1,0)$	$[-\frac{3}{2}, \frac{3}{2}, \frac{1}{2}]$

Table 1: Basic branes for the $SU(5)$ model

group $U(5) \times U(1)$ is summarized in Table 2. The intersection of U_5 with U_1 is non-chiral since $I_{U_5 U_1}$ vanishes. The corresponding non-chiral massless spectrum shown in the table consists of four pairs of $\mathbf{5} + \bar{\mathbf{5}}$ and will be discussed in section 3.7.

$SU(5) \times U(1)^2$	number
$(\mathbf{10}; 2, 0)$	3
$(\mathbf{5}; 1, 1)$	-3
$(\bar{\mathbf{5}}; -1, 1)$	$4-4$

Table 2: Massless spectrum

3.2 Non-chiral stacks

So far, we have obtained the gauge and matter chiral spectrum of the $SU(5)$ GUT using two stacks of magnetized branes. However, in order to complete the model and stabilize all moduli, one needs to add additional stacks of magnetized branes. This has to be done in a manner such that the supersymmetries of all the brane stacks are mutually compatible. To this end, we first examine whether the first two stacks U_5 and U_1 can have mutually compatible supersymmetry in a way suitable for moduli stabilization. The Kähler moduli stabilization conditions are written in eqs. (2.11) and (2.19), corresponding to the cases where charged scalar VEVs are respectively zero or non-zero.

Since the VEV of any charged scalar on the U_5 stack is required to be zero, in order to preserve the gauge symmetry, the supersymmetry conditions for the U_5 stack read:

$$\frac{3}{8} - \frac{1}{2}(J_1 J_2 - 3 J_2 J_3 - J_1 J_3) = 0, \tag{3.13}$$

$$J_1J_2J_3 - \frac{1}{4}(-J_1 - 3J_2 + 3J_3) > 0, \tag{3.14}$$

where we have used the fact that all windings are equal to unity and that eventually the Kähler moduli are stabilized according to our ansatz (3.2), such that $J_{i\bar{j}} = 0$ for $i \neq j$, and we have also defined

$$J_{i\bar{i}} \equiv J_i. \tag{3.15}$$

For the U_1 stack on the other hand, one has the option of turning on a charged scalar VEV without breaking $SU(5)$ gauge invariance. However, since all windings are equal to unity, there are no charged states under $U(1)$ which are $SU(5)$ singlets. Indeed, there is no antisymmetric representation for $U(1)$, while symmetric representations are absent because of our winding choice. The only charged states then come from the intersection of U_1 with U_5 (or its image). Thus, the supersymmetry condition for the U_1 stack follows from eq. (2.11), with the fluxes given in eq. (3.12) and Table 1:

$$-\frac{9}{8} - \frac{1}{2}(J_1J_2 - 3J_2J_3 + 3J_1J_3) = 0, \tag{3.16}$$

$$J_1J_2J_3 - \frac{1}{4}(3J_1 - 3J_2 - 9J_3) > 0. \tag{3.17}$$

Subtracting eq. (3.16) from eq. (3.13) one obtains: $J_1J_3 = -\frac{3}{4}$ which is clearly not allowed. We then conclude that the U_1 stack is not suitable for closed string moduli stabilization without charged scalar VEVs from its intersection with other brane stacks (besides U_5). We therefore need eight new $U(1)$ stacks for stabilizing all the nine geometric Kähler moduli, in the absence of open string VEVs.

In order to find such new stacks, one needs to impose the condition that any chiral fermions, other than those discussed in section 3.1, are $SU(5)$ singlets and thus belong to the 'hidden sector', satisfying:

$$I_{U_5a} + I_{U_5a^*} = 0, \quad \text{for } a = 1,..,8\,. \tag{3.18}$$

We then introduce eight new stacks $O_1, \ldots, O_8$, which carry in general both oblique and diagonal fluxes in order to stabilize eight of the geometric Kähler moduli, using the supersymmetry constraints (2.11). The remaining one is stabilized by the stack U_5. More precisely, to determine the brane stacks $O_1, \ldots, O_8$, we start with our ansatz for both

Kähler and complex structure moduli, and use them to find out the allowed fluxes, consistent with zero net chirality and supersymmetry. Later on, we use the resulting fluxes to show the complete stabilization of moduli, and thus prove the validity of our ansatz.

In general, along a stack-a, the fluxes can be denoted by 3×3 Hermitian matrices,

$$F^a_{(1,1)} = \begin{pmatrix} f_1 & a & b \\ a^* & f_2 & c \\ b^* & c^* & f_3 \end{pmatrix}, \tag{3.19}$$

with f_i's being real numbers, and we have suppressed the superscript 'a' on the matrix components in the rhs of eq. (3.19). The relationships between the matrix elements $(F^a_{(1,1)})_{i\bar{j}}$ and the flux components $p^a_{x^ix^j}$, $p^a_{x^iy^j}$, $p^a_{y^iy^j}$ are:

$$f_i = p_{x^iy^i}\,, \quad a = p_{x^1y^2} + ip_{x^1x^2}\,, \quad b = p_{x^1y^3} + ip_{x^1x^3}\,, \quad c = p_{x^2y^3} + ip_{x^2x^3}\,. \tag{3.20}$$

The subscript $(1,1)$ will also sometimes be suppressed for notational simplicity. We now solve the non-chirality condition (3.18) that a general flux of the type (3.19) must satisfy:

$$I_{U_5a} + I_{U_5a^*} = \det(F^{U_5} - F^a) + \det(F^{U_5} + F^a) = 0\,. \tag{3.21}$$

The general solution for the flux (3.19) is:

$$\frac{3}{4} + (f_1f_2 - 3f_2f_3 - f_1f_3) + (3cc^* - aa^* + bb^*) = 0. \tag{3.22}$$

All additional stacks, including $O_1, \ldots, O_8$, are required to satisfy this condition.

3.3 Supersymmetry constraint

We now impose an additional requirement on the fluxes along the stacks $O_1, \ldots, O_8$, that together with the stack U_5 they should satisfy the supersymmetry conditions (2.11), in the absence of charged scalar VEVs. Using F^a of eq. (3.19), the supersymmetry equations analogous to (3.13) and (3.14) for a stack O_a read:

$$(f_1f_2f_3 - cc^*f_1 - bb^*f_2 - aa^*f_3 + a^*bc^* + ab^*c) - (J_1J_2f_3 + J_2J_3f_1 + J_1J_3f_2) \;=\; 0, \tag{3.23}$$

$$J_1J_2J_3 - [J_1(f_2f_3 - cc^*) + J_2(f_3f_1 - bb^*) + J_3(f_1f_2 - aa^*)] \;>\; 0. \tag{3.24}$$

Next, we obtain two sets of fluxes of the form (3.19) which satisfy eqs. (3.22) and (3.23). The two sets, $O_1, \ldots, O_4$ and $O_5, \ldots, O_8$, are characterized by the diagonal entries in the matrix F^a (3.19), which will be the same for the branes of each set. The motivation behind such choices is dictated by the fact that once the off diagonal components of $J_{i\bar{j}}$ are fixed to zero, these two sets of fluxes along the diagonal, together with the flux of U_5 stack, determine the three diagonal elements J_i (3.15), completely.

3.4 Solution for the stacks $O_1, \ldots, O_4$

In order to find a constraint on the flux components f_1, f_2, f_3 and a, b, c arising out of the requirement that equations (3.13) and (3.23) should be satisfied simultaneously, we start with a particular one-parameter solution of eq. (3.13):

$$J_1 = \frac{3}{4\epsilon^2}, \quad J_2 = \frac{1}{2\epsilon} + \frac{1}{2}, \quad J_3 = \frac{1}{2\epsilon} - \frac{1}{2} \tag{3.25}$$

for arbitrary parameter $\epsilon \in (0, 1)$.[5] Then, by inserting (3.25) into eq. (3.23), one obtains the relation:

$$\frac{3}{4\epsilon^3}\left(\frac{f_2 + f_3}{2}\right) + \frac{1}{4\epsilon^2}\left[\frac{3}{2}(f_3 - f_2) + f_1\right]$$
$$= (f_1 f_2 f_3 - cc^* f_1 - bb^* f_2 - aa^* f_3 + a^* bc^* + ab^* c) + \frac{f_1}{4}\,. \tag{3.26}$$

In solving eqs. (3.22) and (3.26), satisfying also the positivity condition (3.24), we have to keep in mind that f_i's take half-integer values due to the NS-NS B-field background (2.28). On the other hand the parameters a, b, c must be integers, since the windings are all one and there is no B-field turned on along any off-diagonal 2-cycle. Our approach is then to first look for a solution of eq. (3.22) and then examine whether such a solution gives an ϵ from eq. (3.26) such that all the J_i's in eq. (3.25) are positive. In addition, both positivity conditions (3.14) and (3.24) have to be satisfied.

To solve eq. (3.22), we impose the relation $f_2 = -f_3$. The two equations (3.22) and

[5] One can also write down a full two-parameter solution of eq. (3.13), however we prefer to use two different one-parameter families with appropriate parametrization for convenience in model building. The second one-parameter solution will be used in section 3.5.

(3.26) are then reduced to

$$\frac{3}{4} + 2f_1 f_2 + 3f_2^2 + 3cc^* + bb^* - aa^* = 0, \tag{3.27}$$

and

$$\frac{1}{4\epsilon^2}(-3f_2 + f_1) = -f_1 f_2^2 - cc^* f_1 - bb^* f_2 + aa^* f_2 + a^* bc^* + ab^* c + \frac{f_1}{4}. \tag{3.28}$$

A solution of eq. (3.27) with purely real flux components is found to be:

$$f_1 = \frac{5}{2}\,, \quad f_2 = \frac{1}{2}\,, \quad f_3 = -\frac{1}{2}\,, \quad a = 4\,, \quad b = 3\,, \quad c = 1\,. \tag{3.29}$$

Moreover, we notice from eqs. (3.27), (3.28) and the identity:

$$a^* bc^* + ab^* c = 2a_1(b_1 c_1 + b_2 c_2) + 2a_2(b_2 c_1 - b_1 c_2)\,, \tag{3.30}$$

with $a = a_1 + ia_2$, $b = b_1 + ib_2$, $c = c_1 + ic_2$, that other solutions can be found simply by replacing some of the real components of a, b, c by imaginary ones modulo signs, as long as the values of the products aa^*, bb^*, cc^*, as well as that of $(a^* bc^* + ab^* c)$ remain unchanged. We make use of such choices for canceling off-diagonal $D5$-brane tadpoles which for a general flux matrix (3.19) read (using eq. (2.24)):

$$\begin{aligned} Q^{5,a}_{1\bar{1}} &= (f_2 f_3 - cc^*)\,, \quad Q^{5,a}_{2\bar{2}} = (f_3 f_1 - bb^*)\,, \quad Q^{5,a}_{3\bar{3}} = (f_1 f_2 - aa^*)\,, \\ Q^{5,a}_{1\bar{2}} &= (b^* c - a^* f_3)\,, \quad Q^{5,a}_{2\bar{3}} = (b^* a - c^* f_1)\,, \quad Q^{5,a}_{3\bar{1}} = (ac - bf_2)\,. \end{aligned} \tag{3.31}$$

Here we have used the complex coordinates $z^i, \bar{z}^i$ and the assumption that complex structure is eventually stabilized as in eq. (1.3).

The result of our analysis above, giving fluxes for the brane stacks $O_1, \ldots, O_4$, (including the solution (3.29)) is presented in Appendix A, in eqs. (A.2), (A.7), (A.12), (A.17). In this Appendix, we also show that the net chiral fermion contribution from the intersection of each of the four stacks $O_1, \ldots, O_4$ with U_5 (and its image) is zero, as shown in eqs. (A.3), (A.8), (A.13), (A.18). Oblique tadpoles $Q^5_{1\bar{2}}$, $Q^5_{2\bar{3}}$, $Q^5_{3\bar{1}}$ are given in eqs. (A.4), (A.9), (A.14), (A.19) and their cancellations among these branes is also apparent. This leaves only diagonal $D5$ tadpoles, given in eqs. (A.5), (A.10), (A.15), (A.20). The fluxes in real basis are given in eqs. (A.6), (A.11), (A.16), (A.21). In Table 3, we summarize

all Chern numbers and windings for the stacks $O_1, \dots, O_4$, as well as those for the stacks $O_5, \dots, O_8$ appearing in the next subsection.

From eqs. (3.23) and (3.28), the stacks $O_1, \dots, O_4$ satisfy the supersymmetry condition:

$$\frac{195}{8} - \frac{1}{2}[-J_1 J_2 + 5 J_2 J_3 + J_1 J_3] = 0, \tag{3.32}$$

for $\epsilon = \frac{1}{10}$ in eq. (3.25). The positivity condition (3.24) for all of them has the following final form:

$$J_1 J_2 J_3 + \frac{5}{4} J_1 + \frac{41}{4} J_2 + \frac{59}{4} J_3 > 0, \tag{3.33}$$

which is obviously satisfied for the solution (3.25) with $\epsilon = \frac{1}{10}$. Also, the chiral fermion degeneracies on the intersections $U_5 - O_a$ and $U_5 - O_a^*$ are equal to

$$I_{U_5 O_a} = 23\,, \quad I_{U_5 O_a^*} = -23\,, \quad a = 1, \dots, 4\,, \tag{3.34}$$

giving vanishing net chirality for all of them individually. The non-trivial tadpole contributions from the four stacks are:

$$Q^9 = 4\,, \quad Q^5_{x^1 y^1} = -5\,, \quad Q^5_{x^2 y^2} = -41\,, \quad Q^5_{x^3 y^3} = -59\,. \tag{3.35}$$

3.5 Additional stacks: $O_5, \dots, O_8$

In the last subsection we found four stacks $O_1, \dots, O_4$ with oblique fluxes but diagonal 5-brane charges. Clearly, in order to stabilize all the Kähler moduli, we need at least four additional stacks with oblique fluxes. The search for such branes is simplified by observing that the supersymmetry condition (3.13) for the stack U_5 has another one parameter family of solutions, independent of (3.25), which solves also the condition (3.32) for the stacks $O_1, \dots, O_4$:

$$J_1 = \frac{300\alpha}{4\alpha^2 - 99}\,, \quad J_2 = \alpha\,, \quad J_3 = \frac{99}{4\alpha}\,, \quad \text{with} \quad \alpha^2 > \frac{99}{4}\,. \tag{3.36}$$

By inserting expressions (3.36) into the general supersymmetry condition (3.23), and following steps similar to those of the last subsection, we find the set of stacks $O_5, \dots, O_8$ given in Appendix A, with fluxes as in eqs. (A.22), (A.27), (A.32), (A.37). One of these solutions has flux components:

$$f_1 = -\frac{25}{2}\,, \quad f_2 = \frac{1}{2}\,, \quad f_3 = \frac{1}{2}\,, \quad a = -2i\,, \quad b = -i\,, \quad c = 1\,, \tag{3.37}$$

while the others can be obtained by trivial changes of the off-diagonal elements, as for the stacks $O_1, \ldots, O_4$ discussed in the previous subsection. Oblique $D5$ tadpoles are written in eqs. (A.24), (A.29), (A.34), (A.39) and the diagonal ones in eqs. (A.25), (A.30), (A.35), (A.40). The net $SU(5)$ non-singlet fermion chirality for these stacks is also zero, as shown in eqs. (A.23), (A.28), (A.33), (A.38). Once again, all off-diagonal $D5$ tadpoles of the type $Q^5_{1\bar{2}}$, $Q^5_{2\bar{3}}$ and $Q^5_{3\bar{1}}$ cancel among the contributions of the four brane stacks. In Table 3, we summarize the Chern numbers and windings of the stacks $O_5, \ldots, O_8$, as well.

The four stacks $O_5, \ldots, O_8$ satisfy the supersymmetry condition:

$$\frac{87}{8} - \frac{1}{2}[J_1 J_2 - 25 J_2 J_3 + J_1 J_3] = 0, \tag{3.38}$$

for

$$\alpha^2 = \frac{99}{4} \times \frac{1431}{1131}, \tag{3.39}$$

consistently with the inequality (3.36). For this value of α, the positivity conditions (3.14) and (3.17) for the U_5 and U_1 stacks are also satisfied by J_i's of the form (3.36). On the other hand, using the flux components (3.19) from Table 3, the positivity condition for the four new stacks takes the following form:

$$J_1 J_2 J_3 + \frac{3}{4} J_1 + \frac{29}{4} J_2 + \frac{41}{4} J_3 > 0, \tag{3.40}$$

and is again obviously satisfied, as is the positivity condition (3.33) for stacks $O_1, \ldots, O_4$. The final uncanceled tadpoles from these stacks are:

$$Q^9 = 4\,, \quad Q^5_{x^1 y^1} = -3\,, \quad Q^5_{x^2 y^2} = -29\,, \quad Q^5_{x^3 y^3} = -41\,, \tag{3.41}$$

while the chiral fermion degeneracy from the intersections $U_5 - O_a$ and $U_5 - O_a^*$ is given by:

$$I_{U_5 O_a} = 14\,, \quad I_{U_5 O_a^*} = -14\,, \quad a = 5, \ldots, 8\,. \tag{3.42}$$

3.6 Tadpole cancellation

We now collect the tadpole contribution from different stacks to find out how the total RR charges cancel in our model by adding two extra stacks of single branes, A and B. The

Stack	No. of branes: N_{O_a}	Windings $(n_{x^1}^{O_a}, n_{x^2}^{O_a}, n_{x^3}^{O_a})$ $(n_{y^1}^{O_a}, n_{y^2}^{O_a}, n_{y^3}^{O_a})$	Diag. Chern no. $(m_{x^1y^1}^{O_a}, m_{x^2y^2}^{O_a}, m_{x^3y^3}^{O_a})$	Diagonal fluxes $[f_1^a, f_2^a, f_3^a]$	Oblique Chern no.
O_1	1	$(1,1,1)$ $(1,1,1)$	(2,0,-1)	$[\frac{5}{2},\frac{1}{2},-\frac{1}{2}]$	$m_{x^1y^2}^{O_1} = m_{x^2y^1}^{O_1} = 4$ $m_{x^1y^3}^{O_1} = m_{x^3y^1}^{O_1} = 3$ $m_{x^2y^3}^{O_1} = m_{x^3y^2}^{O_1} = 1$
O_2	1	$(1,1,1)$ $(1,1,1)$	(2,0,-1)	$[\frac{5}{2},\frac{1}{2},-\frac{1}{2}]$	$m_{x^1y^2}^{O_2} = m_{x^2y^1}^{O_2} = 4$ $m_{x^1y^3}^{O_2} = m_{x^3y^1}^{O_2} = -3$ $m_{x^2y^3}^{O_2} = m_{x^3y^2}^{O_2} = -1$
O_3	1	$(1,1,1)$ $(1,1,1)$	(2,0,-1)	$[\frac{5}{2},\frac{1}{2},-\frac{1}{2}]$	$m_{x^1y^2}^{O_3} = m_{x^2y^1}^{O_3} = -4$ $m_{x^3x^1}^{O_3} = m_{y^3y^1}^{O_3} = 3$ $m_{x^2x^3}^{O_3} = m_{y^2y^3}^{O_3} = 1$
O_4	1	$(1,1,1)$ $(1,1,1)$	(2,0,-1)	$[\frac{5}{2},\frac{1}{2},-\frac{1}{2}]$	$m_{x^1y^2}^{O_4} = m_{x^2y^1}^{O_4} = -4$ $m_{x^3x^1}^{O_4} = m_{y^3y^1}^{O_4} = -3$ $m_{x^2x^3}^{O_4} = m_{y^2y^3}^{O_4} = -1$
O_5	1	$(1,1,1)$ $(1,1,1)$	(-13,0,0)	$[\frac{-25}{2},\frac{1}{2},\frac{1}{2}]$	$m_{x^1x^2}^{O_5} = m_{y^1y^2}^{O_5} = -2$ $m_{x^3x^1}^{O_5} = m_{y^3y^1}^{O_5} = 1$ $m_{x^2y^3}^{O_5} = m_{x^3y^2}^{O_5} = 1$
O_6	1	$(1,1,1)$ $(1,1,1)$	(-13,0,0)	$[\frac{-25}{2},\frac{1}{2},\frac{1}{2}]$	$m_{x^1x^2}^{O_6} = m_{y^1y^2}^{O_6} = -2$ $m_{x^3x^1}^{O_6} = m_{y^3y^1}^{O_6} = -1$ $m_{x^2y^3}^{O_6} = m_{x^3y^2}^{O_6} = -1$
O_7	1	$(1,1,1)$ $(1,1,1)$	(-13,0,0)	$[\frac{-25}{2},\frac{1}{2},\frac{1}{2}]$	$m_{x^1x^2}^{O_7} = m_{y^1y^2}^{O_7} = 2$ $m_{x^1y^3}^{O_7} = m_{x^3y^1}^{O_7} = -1$ $m_{x^2x^3}^{O_7} = m_{y^2y^3}^{O_7} = 1$
O_8	1	$(1,1,1)$ $(1,1,1)$	(-13,0,0)	$[\frac{-25}{2},\frac{1}{2},\frac{1}{2}]$	$m_{x^1x^2}^{O_8} = m_{y^1y^2}^{O_8} = 2$ $m_{x^1y^3}^{O_8} = m_{x^3y^1}^{O_8} = 1$ $m_{x^2x^3}^{O_8} = m_{y^2y^3}^{O_8} = -1$

Table 3: Chern numbers and windings of the oblique stacks $O_1, \ldots, O_8$

Stack no. a	No. of branes: N_a	Windings $(\hat{n}_1^a, \hat{n}_2^a, \hat{n}_3^a)$	Chern no. $(\ \hat{m}_1^a, \hat{m}_2^a, \hat{m}_3^a\ )$	Fluxes $\left[\frac{(\hat{m}_1^a+\hat{n}_1^a/2)}{\hat{n}_1^a}, \frac{(\hat{m}_2^a+\hat{n}_2^a/2)}{\hat{n}_2^a}, \frac{(\hat{m}_3^a+\hat{n}_3^a/2)}{\hat{n}_3^a}\right]$
Stack-A	1	$(1,1,1)$	$(147,0,0)$	$[\frac{295}{2}, \frac{1}{2}, \frac{1}{2}\ ]$
Stack-B	1	$(1,1,1)$	$(1,16,0)$	$[\frac{3}{2}, \frac{33}{2}, \frac{1}{2}]$

Table 4: A and B branes

tadpole contributions from stacks $O_1, \ldots, O_4$ with oblique fluxes, are given in eq. (3.35), while those from stacks $O_5, \ldots O_8$ are given in eq. (3.41). In addition, the stacks U_5 and U_1 together contribute:

$$Q^9 = 6\,, \quad Q^5_{x^1y^1} = -\frac{1}{2}\,, \quad Q^5_{x^2y^2} = -\frac{9}{2}\,, \quad Q^5_{x^3y^3} = \frac{3}{2}\,, \tag{3.43}$$

where we used the flux components (3.6) and (3.12). These tadpoles are then saturated by the brane stacks A and B of Table 4. Their contributions to the tadpoles are:

$$Q^9 = 2\,, \quad Q^5_{x^1y^1} = \frac{34}{4}\,, \quad Q^5_{x^2y^2} = \frac{298}{4}\,, \quad Q^5_{x^3y^3} = \frac{394}{4}\,, \tag{3.44}$$

which precisely cancel the contributions from eqs. (3.35), (3.41) and (3.43). Moreover, chiral fermion multiplicities from the intersections of stacks A and B with U_5 vanish, as well:

$$I_{U_5A} = I_{U_5A^*} = I_{U_5B} = I_{U_5B^*} = 0\,. \tag{3.45}$$

We have thus obtained fluxes for the twelve stacks, saturating both $D9$ and $D5$ tadpoles. However, for supersymmetry, we have only discussed the conditions for nine of the twelve brane stacks, namely U_5 and $O_1, \ldots, O_8$. The status of supersymmetry for the brane stacks U_1, A and B will be studied later, in section 5.

Before closing this section, we also examine briefly whether it would be possible to manage tadpole cancellation without adding the extra stacks A and B, within the context of our construction specified by the choice (3.11) of intersection numbers. Note that the nine stacks U_5 and $O_1, \ldots, O_8$ were the minimal ones needed for Kähler moduli stabilization, since the use of the U_1 brane for this purpose was ruled out, as we discussed in section 3.2. The U_1 stack on the other hand is needed to get the right $SU(5)$ particle spectrum. Thus,

in order to avoid the use of stacks A and B, one needs to examine whether there are solutions, other than the one found in eq. (3.12), for fluxes along the stack-U_1 such that tadpole cancellations are possible, while a scalar VEV charged under this $U(1)$ may have to be turned on in order to maintain supersymmetry. In such a situation, one needs a winding number three ($\det W = 3$) for the stack U_1 to saturate the $D9$ tadpole. Moreover, all oblique fluxes along the U_1 stack have to vanish, otherwise they would give rise to uncanceled tadpoles in oblique directions. Then, by writing the tadpole contributions of three diagonal fluxes f_i satisfying the constraint (3.11), it can be easily seen that one is not able to cancel the combined tadpoles from stacks U_5 and $O_1, \ldots, O_8$. Such a possibility is therefore ruled out. Of course, one could try to find a solution that satisfies the constraint (3.11) but not necessarily (3.8), as we discussed already in section 3.1. Alternatively, one can possibly attempt to manage with just two stacks U_1 and A, by using winding number two in one of them. These are straight-forward exercises for the interested reader who would like to examine these cases.

3.7 Non-chiral spectrum

The degeneracies of non-chiral states coming from intersections of the stack U_5 with O_a and O_a^* are already given in eqs. (3.34) and (3.42), leading to $4 \times (23 + 14) = 148$ pairs of $(\mathbf{5} + \bar{\mathbf{5}})$ representations of $SU(5)$. They follow from the degeneracy formulae (2.30), when the net numbers of left- and right-handed fermions are equal. In our case, this is insured since $I_{U_5 O_a} = -I_{U_5 O_a^*}$. However, non-chiral particle spectrum also follows from eqs. (2.30), (2.31) and (2.32), when any of I_{ab}, I_{ab^*}, $I_{aa^*}^A$ and $I_{aa^*}^S$ are zero, as explained at the end of section 2.4. This occurs because for instance $\prod_i (\tilde{\hat{m}}_i^a \hat{n}_i^b \pm \hat{n}_i^a \tilde{\hat{m}}_i^b)$ vanishes along one or more of the 2-tori, T_j^2. Similarly for $I_{aa^*}^A$ or $I_{aa^*}^S$, this occurs because of the vanishing of fluxes along one or more of the T^2's. Given the fluxes in stack U_5, which are non-zero along all three T^2's, non-chiral states can come only from various intersections of the U_5 stack with other branes.

For example, the intersection numbers between stacks U_5 and U_1 are given in eq. (3.11). One sees that $I_{U_5 U_1}$ is zero as $(\tilde{\hat{m}}_i^{U_5} \hat{n}_i^{U_1} - \hat{n}_i^{U_5} \tilde{\hat{m}}_i^{U_1})$ vanishes along T_1^2 and T_3^2. However, in this case there exists a non-zero intersection number in $d = 8$ dimensions corresponding

to the T_2^2 compactification of the $d = 10$ theory, given by:

$$I_{U_5U_1}|_{T_1^2,T_3^2} = (\tilde{\hat{m}}_2^{U_5}\hat{n}_2^{U_1} - \hat{n}_2^{U_5}\tilde{\hat{m}}_2^{U_1}) = -2, \tag{3.46}$$

with the subscripts T_1^2, T_3^2 of $I_{U_5U_1}|$ standing for those tori along which the intersection number vanishes. This implies two negative chirality (right-handed) fermions in $d = 8$, in the fundamental representation of $SU(5)$. Under further compactification along T_1^2 and T_3^2, we get four Dirac spinors in $d = 4$, or equivalently four pairs of $(\mathbf{5} + \bar{\mathbf{5}})$ Weyl fermions, shown already in the massless spectrum of Table 2. They give rise to four pairs of electroweak higgses, having non-vanishing tree-level Yukawa couplings with the down-type quarks and leptons, as it can be easily seen.

A similar analysis for the remaining stacks A and B gives chiral spectra in $d = 6$ with degeneracies:

$$I_{U_5A}|_{T_3^2} = (\tilde{\hat{m}}_1^{U_5}\hat{n}_1^{A} - \hat{n}_1^{U_5}\tilde{\hat{m}}_1^{A}) \times (\tilde{\hat{m}}_2^{U_5}\hat{n}_2^{A} - \hat{n}_2^{U_5}\tilde{\hat{m}}_2^{A}) = 149\,, \tag{3.47}$$

and

$$I_{U_5A^*}|_{T_2^2} = (\tilde{\hat{m}}_1^{U_5}\hat{n}_1^{A} + \hat{n}_1^{U_5}\tilde{\hat{m}}_1^{A}) \times (\tilde{\hat{m}}_2^{U_5}\hat{n}_2^{A} + \hat{n}_2^{U_5}\tilde{\hat{m}}_2^{A}) = 146\,. \tag{3.48}$$

They give rise to $149 + 146 = 295$ pairs of $(\mathbf{5} + \bar{\mathbf{5}})$. Similarly, we obtain for the stack B:

$$I_{U_5B}|_{T_3^2} = (\tilde{\hat{m}}_1^{U_5}\hat{n}_1^{B} - \hat{n}_1^{U_5}\tilde{\hat{m}}_1^{B}) \times (\tilde{\hat{m}}_2^{U_5}\hat{n}_2^{B} - \hat{n}_2^{U_5}\tilde{\hat{m}}_2^{B}) = 51\,, \tag{3.49}$$

and

$$I_{U_5B^*}|_{T_1^2} = (\tilde{\hat{m}}_2^{U_5}\hat{n}_2^{B} + \hat{n}_2^{U_5}\tilde{\hat{m}}_2^{B}) \times (\tilde{\hat{m}}_3^{U_5}\hat{n}_3^{B} + \hat{n}_3^{U_5}\tilde{\hat{m}}_3^{B}) = 16\,, \tag{3.50}$$

leading to $51 + 16 = 67$ pairs of $(\mathbf{5} + \bar{\mathbf{5}})$. All these non chiral states become massive by displacing appropriately the branes A and B in directions along the tori T_3^2, T_2^2 and T_3^2, T_1^2, respectively.

In addition to the states above, there are several $SU(5)$ singlets coming from the intersections among the branes $O_1, \ldots, O_8$, U_1, A and B. Since they do not play any particular role in physics concerning our analysis, we do not discuss them explicitly here. However, such scalars from the non-chiral intersections among U_1, A and B will be used in section 5 for supersymmetrizing these stacks, by cancelling the corresponding non-zero FI parameters upon turning on non-trivial VEVs for these fields. The corresponding non-chiral spectrum will be therefore discussed below, in section 5.

4 Moduli stabilization

Earlier, we have found fluxes along the nine brane stacks $U_5, O_1, \ldots, O_8$, given in Tables 1, 2, 3, 4 and in Appendix A, consistent with our ansatz (1.3) for the complex structure and (3.2) for the geometric Kähler moduli. We now prove our ansatz by showing that both τ and J are uniquely fixed to the values (1.3), (3.2) and (3.36), (3.39). To show this, we make use of the full supersymmetry conditions for the U_5 stack as well as for the stacks $O_1, \ldots, O_8$.

For the complex structure moduli stabilization, we make use of the $F^a_{(2,0)}$ condition (2.15) implying that purely holomorphic components of fluxes vanish. Then, by inserting the flux components $p_{x^ix^j}$, $p_{x^iy^j}$ $p_{y^iy^j}$, as given in Table 1 and Table 3, as well as in Appendix A, along the U_5 and $O_1, .., O_8$ stacks, we obtain a set of conditions on the complex structure matrix τ, given explicitly in Appendix B in eqs. (B.1) - (B.47). These equations imply the final answer (1.3). The details can be found in Appendix B.

For Kähler moduli stabilization, we make use of the D-flatness condition in stacks U_5, $O_1, \ldots O_8$ which amounts to using the last two equations in (2.11). Explicit stabilization of the geometric Kähler moduli to the diagonal form, $J_{i\bar{j}} = 0$, $(i \neq j)$ is given in eqs. (C.2) - (C.26) of Appendix C. For the stabilization of the diagonal components, the relevant equations are: (3.13), (3.14), (3.32), (3.33), (3.38), (3.40). The final solution for the stabilized moduli is given in eqs. (3.36) and (3.39). The numerical values of J_i's can also be approximated as:

$$J_1 \sim 63.96\,, \quad J_2 \sim 5.59\,, \quad J_3 \sim 4.42\,. \tag{4.1}$$

5 Supersymmetry of stacks U_1, A and B

We now discuss the supersymmetry of the remaining stacks U_1, A and B by making use of the D-flatness conditions (2.18), (2.19) and (2.20). From these equations, suppressing the superscript a, we obtain the FI parameters ξ as:

$$\xi = \frac{F^3_{(1,1)} - J^2 F_{(1,1)}}{J^3 - J F^2_{(1,1)}}\,, \tag{5.1}$$

where we have made use of eq. (2.14) and the canonical volume normalization (1.4). Then, using the values of the magnetic fluxes in stacks U_1, A and B from Tables 1 and 4, the explicit form of the FI parameters in terms of the moduli J_i (that are already completely fixed to the values (4.1)) is given by:

$$\xi^{U_1} = \frac{-\frac{9}{8} - \frac{1}{2}(J_1 J_2 - 3J_2 J_3 + 3J_1 J_3)}{J_1 J_2 J_3 - \frac{1}{4}(3J_1 - 3J_2 - 9J_3)}\,, \tag{5.2}$$

$$\xi^{A} = \frac{\frac{295}{8} - \frac{1}{2}(J_1 J_2 + 295J_2 J_3 + J_1 J_3)}{J_1 J_2 J_3 - \frac{1}{4}(J_1 + 295J_2 + 295J_3)}\,, \tag{5.3}$$

$$\xi^{B} = \frac{\frac{33}{8} - \frac{1}{2}(J_1 J_2 + 3J_2 J_3 + 33J_1 J_3)}{J_1 J_2 J_3 - \frac{1}{4}(33J_1 + 3J_2 + 99J_3)}\,, \tag{5.4}$$

leading to the numerical values:

$$\xi^{U_1} \sim -0.366\,, \quad \xi^{A} \sim -4.753\,, \quad \xi^{B} \sim -5.173\,. \tag{5.5}$$

On the other hand, the charged scalar VEVs v_ϕ entering in the modified D-flatness conditions (2.18) and (2.19) are related to the modified FI parameters ξ^a/G^a, as it can be easily seen from the expressions (2.16) and (2.17), that are also relevant for the perturbativity criterion: $v_\phi << 1$ in string units. Their knowledge needs determination of the matter field metric G^a on the branes U_1, A and B. For this purpose, we make use of eq. (2.22) with the angles θ_i defined in eq. (2.21). One finds the following values for the metric G in the three stacks:

$$G^{U_1} \sim 2.815\ , \quad G^{A} \sim 50.45\ , \quad G^{B} \sim 94.551\,, \tag{5.6}$$

that lead to the modified FI parameters:

$$\frac{\xi^{U_1}}{G^{U_1}} \sim -0.130\ , \quad \frac{\xi^{A}}{G^{A}} \sim -0.094\ , \quad \frac{\xi^{B}}{G^{B}} \sim -0.057\,. \tag{5.7}$$

Note that the positivity conditions (2.20), giving positive gauge couplings through eq. (2.17) for the stacks U_1, A and B, hold as well. These expressions appear also in the FI parameters ξ^a as the denominators in the rhs of eqs. (5.2) - (5.4).

The last part of the exercise is to cancel the FI parameters (5.7) with VEVs of specific charged scalars living on the branes U_1, A and B, in order to satisfy the D-flatness condition

(2.18). For this we first compute the chiral fermion multiplicities on their intersections:

$$I_{U_1A} = (F^{U_1} - F^A)^3 = 0 \ , \quad I_{U_1B} = (F^{U_1} - F^B)^3 = 0 \ , \quad I_{AB} = (F^A - F^B)^3 = 0 \,. \quad (5.8)$$

Since they all vanish, there are equal numbers of chiral and anti-chiral fields in each of these intersections. In order to determine separately their multiplicities, we follow the method used in section 3.7 and compute:

$$I_{U_1A}|_{T_3^2} = -149 \ , \quad I_{U_1B}|_{T_3^2} = 45 \ , \quad I_{AB}|_{T_3^2} = -2336 \,. \qquad (5.9)$$

These correspond to chiral fermion multiplicities in six dimensions generating upon compactification to $d = 4$ pairs of left- and right-handed fermions. We also have:

$$I_{U_1A^*} = (F^{U_1} + F^A)^3 = 292 \ , \quad I_{U_1B^*} = (F^{U_1} + F^B)^3 = 0 \ , \quad I_{AB^*} = (F^A + F^B)^3 = 149 \times 17 \,, \qquad (5.10)$$

which gives zero net chirality for the $U_1 - B^*$ intersection. Computing

$$I_{U_1B^*}|_{T_1^2} = 18 \,, \qquad (5.11)$$

one then finds 18 pairs of left- and right-handed fermions in $d = 4$ from this intersection.

As a result, we have the following non-chiral fields, where the superscript refers to the two stacks between which the open string is stretched and the subscript denotes the charges under the respective $U(1)$'s : $(\phi_{+-}^{U_1A},\ \phi_{-+}^{U_1A})$, $(\phi_{+-}^{U_1B},\ \phi_{-+}^{U_1B})$, $(\phi_{+-}^{AB},\ \phi_{-+}^{AB})$, $(\phi_{++}^{U_1B^*},\ \phi_{--}^{U_1B^*})$, with fields in the brackets having multiplicities 149, 45, 2336 and 18, respectively. Restricting only to possible VEVs for these fields, eq. (2.18) takes the following form for the stacks U_1, A and B:

$$\xi^{U_1}/G^{U_1} + |\phi_{+-}^{U_1A}|^2 - |\phi_{-+}^{U_1A}|^2 + |\phi_{+-}^{U_1B}|^2 - |\phi_{-+}^{U_1B}|^2 + |\phi_{++}^{U_1B^*}|^2 - |\phi_{--}^{U_1B^*}|^2 = 0 \,, \qquad (5.12)$$

$$\xi^{A}/G^{A} + |\phi_{-+}^{U_1A}|^2 - |\phi_{+-}^{U_1A}|^2 + |\phi_{+-}^{AB}|^2 - |\phi_{-+}^{AB}|^2 = 0 \,, \qquad (5.13)$$

$$\xi^{B}/G^{B} + |\phi_{-+}^{U_1B}|^2 - |\phi_{+-}^{U_1B}|^2 + |\phi_{-+}^{AB}|^2 - |\phi_{+-}^{AB}|^2 + |\phi_{++}^{U_1B^*}|^2 - |\phi_{--}^{U_1B^*}|^2 = 0 \,. \qquad (5.14)$$

These equations can also be written as:

$$\xi^{U_1}/G^{U_1} + (v^{U_1})^2 = 0 \ \Rightarrow \ (v^{U_1})^2 = -(\xi^{U_1}/G^{U_1}) \,, \qquad (5.15)$$

$$\xi^{A}/G^{A} + (v^{A})^2 = 0 \ \Rightarrow \ (v^{A})^2 = -(\xi^{A}/G^{A}) \,, \qquad (5.16)$$

$$\xi^{B}/G^{B} + (v^{B})^2 = 0 \ \Rightarrow \ (v^{B})^2 = -(\xi^{B}/G^{B}) \,, \qquad (5.17)$$

following the notation of eq. (2.19), where we defined:

$$\begin{aligned}(v^{U_1})^2 &= |\phi_{+-}^{U_1A}|^2 - |\phi_{-+}^{U_1A}|^2 + |\phi_{+-}^{U_1B}|^2 - |\phi_{-+}^{U_1B}|^2 + |\phi_{++}^{U_1B^*}|^2 - |\phi_{--}^{U_1B^*}|^2 \\ &\equiv (v^{U_1A})^2 + (v^{U_1B})^2 + (v^{U_1B^*})^2\,, \end{aligned} \tag{5.18}$$

$$\begin{aligned}(v^{A})^2 &= |\phi_{-+}^{U_1A}|^2 - |\phi_{+-}^{U_1A}|^2 + |\phi_{+-}^{AB}|^2 - |\phi_{-+}^{AB}|^2 \\ &\equiv -(v^{U_1A})^2 + (v^{AB})^2\,, \end{aligned} \tag{5.19}$$

$$\begin{aligned}(v^{B})^2 &= |\phi_{-+}^{U_1B}|^2 - |\phi_{+-}^{U_1B}|^2 + |\phi_{-+}^{AB}|^2 - |\phi_{+-}^{AB}|^2 + |\phi_{++}^{U_1B^*}|^2 - |\phi_{--}^{U_1B^*}|^2 \\ &\equiv -(v^{U_1B})^2 - (v^{AB})^2 + (v^{U_1B^*})^2\,, \end{aligned} \tag{5.20}$$

with for instance $(v^{AB})^2 = |\phi_{+-}^{AB}|^2 - |\phi_{-+}^{AB}|^2$ and similarly for the others.

Since we have three equations and four unknowns, we choose to obtain a special solution by setting $(v^{U_1B})^2 = 0$. Equations (5.18) - (5.20) then give:

$$(v^{U_1A})^2 + (v^{U_1B^*})^2 = -(\xi^{U_1}/G^{U_1}) \sim 0.130\,, \tag{5.21}$$

$$-(v^{U_1A})^2 + (v^{AB})^2 = -(\xi^{A}/G^{A}) \sim 0.094\,, \tag{5.22}$$

$$-(v^{AB})^2 + (v^{U_1B^*})^2 = -(\xi^{B}/G^{B}) \sim 0.057\,, \tag{5.23}$$

that can be solved to obtain:

$$(v^{U_1A})^2 = -0.011\,, \quad (v^{U_1B^*})^2 = 0.141\,, \quad (v^{AB})^2 = 0.084\,. \tag{5.24}$$

Recalling from eqs. (5.18) - (5.20) that

$$(v^{U_1A})^2 = |\phi_{+-}^{U_1A}|^2 - |\phi_{-+}^{U_1A}|^2\,, \quad (v^{U_1B^*})^2 = |\phi_{++}^{U_1B^*}|^2 - |\phi_{--}^{U_1B^*}|^2\,, \quad (v^{AB})^2 = |\phi_{+-}^{AB}|^2 - |\phi_{-+}^{AB}|^2\,, \tag{5.25}$$

and comparing with the results of eq. (5.24) (taking into account the different signs), VEVs for the fields $\phi_{-+}^{U_1A}$, $\phi_{++}^{U_1B^*}$ and ϕ_{+-}^{AB} are switched on. Moreover, as required by the validity of the approximation, the values of the charged scalar VEVs satisfy the condition $v^a << 1$ in string units.

6 Conclusions

In conclusion, in this work, we have constructed a three generation $SU(5)$ supersymmetric GUT in simple toroidal compactifications of type I string theory with magnetized $D9$-branes. All 36 closed string moduli are fixed in a $\mathcal{N}=1$ supersymmetric vacuum, apart from the dilaton, in a way that the T^6-torus metric becomes diagonal with the six internal radii given in terms of the integrally quantized magnetic fluxes. Supersymmetry requirement and RR tadpole cancellation conditions impose some of the charged open string scalars (but $SU(5)$ singlets) to acquire non-vanishing VEVs, breaking part of the $U(1)$ factors. The rest become massive by absorbing the RR scalars which are part of the Kähler moduli supermultiplets. Thus, the final gauge group is just $SU(5)$ and the chiral gauge non-singlet spectrum consists of three families with the quantum numbers of quarks and leptons, transforming in the $\mathbf{10}+\bar{\mathbf{5}}$ representations of $SU(5)$. It is of course desirable to study the physics of this model in detail and perhaps to construct other more 'realistic' variations, using the same framework which has an exact string description. Some of the obvious questions to examine are:

1. Give a mass to the non-chiral gauge non-singlet states with the quantum numbers of higgses transforming in pairs of $\mathbf{5}+\bar{\mathbf{5}}$ representations, keeping massless only one pair needed to break the electroweak symmetry. A first partial discussion was given in section 3.7.

2. Break the $SU(5)$ GUT symmetry down to the Standard Model, which can be in principle realized at the string level separating the $U(5)$ stack into $U(3)\times U(2)$ by parallel brane displacement. However, one would like to realize at the same time the so-called doublet-triplet splitting for the Higgs $\mathbf{5}+\bar{\mathbf{5}}$ pair, i.e. giving mass to the unwanted triplets which can mediate fast proton decay and invalidate gauge coupling unification, while keeping the doublets massless. One possibility would be to deform the model by introducing angles, in realizing the $SU(5)$ breaking. In any case, problems (1) and (2) may be related.

3. Compute and study the Yukawa couplings. A general defect of the present construction, already known in the literature, is the absence of up-type Yukawa couplings.

In this respect, some recent progress using D-brane instantons may be useful for up-quark mass generation [20–22].

4. Study the question of supersymmetry breaking. An attractive direction would be to start with a supersymmetry breaking vacuum in the absence of charged scalar VEVs for the extra branes needed to satisfy the RR tadpole cancellation, $U(1) \times U(1)_A \times U(1)_B$ in our construction. This 'hidden sector' could then mediate supersymmetry breaking, which is mainly of D-type, to the Standard Model via gauge interactions. Gauginos can then acquire Dirac masses at one loop without breaking the R-symmetry, due to the extended supersymmetric nature of the gauge sector [23].

Thus, this framework offers a possible self-consistent setup for string phenomenology, in which one can build simple calculable models of particle physics with stabilized moduli and implement low energy supersymmetry breaking that can be studied directly at the string level.

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A Explicit solutions for $O_1, \ldots, O_8$

In this Appendix, we write all the fluxes in the complex coordinate basis $(z, \bar{z})$ with $z = x + iy$. Then, for the windings and 1st Chern numbers of Table 1, we obtain:

$$F^{U_5}_{(1,1)} = -\frac{i}{2} \begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix} \begin{pmatrix} -\frac{3}{2} & & \\ & -\frac{1}{2} & \\ & & \frac{1}{2} \end{pmatrix} \begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix} . \tag{A.1}$$

Below, we sometimes suppress the subscript $(1,1)$ to keep the expressions simpler. The fluxes of the 8 stacks $O_1, \ldots, O_8$ can also be written in the same coordinate basis:

$$F^{O_1}_{(1,1)} = -\frac{i}{2}\begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix}\begin{pmatrix} \frac{5}{2} & 4 & 3 \\ 4 & \frac{1}{2} & 1 \\ 3 & 1 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix}. \tag{A.2}$$

From eq. (A.2) we get

$$|F^{U_5} + F^{O_1}| = 23\,, \quad |F^{U_5} - F^{O_1}| = -23\,, \quad |F^{O_1}| = \frac{195}{8}\,, \tag{A.3}$$

where we have used the notation $|F^{U_5} + F^{O_1}| \equiv \det(F^{U_5} + F^{O_1})$ etc. The oblique $D5$ tadpoles are:

$$Q^{O_1}_{1\bar{2}} = 3 + 2\,, \quad Q^{O_1}_{2\bar{3}} = 12 - \frac{5}{2}\,, \quad Q^{O_1}_{3\bar{1}} = 4 - \frac{3}{2}\,, \tag{A.4}$$

while the diagonal ones are:

$$Q^{O_1}_{1\bar{1}} = -\frac{5}{4}\,, \quad Q^{O_1}_{2\bar{2}} = -\frac{41}{4}\,, \quad Q^{O_1}_{3\bar{3}} = -\frac{59}{4}\,. \tag{A.5}$$

In real coordinates, the fluxes are:

$$p^{O_1}_{x^1y^1} = \frac{5}{2},\; p^{O_1}_{x^2y^2} = -p^{O_1}_{x^3y^3} = \frac{1}{2},\; p^{O_1}_{x^1y^2} = p_{x^2y^1} = 4,\; p^{O_1}_{x^1y^3} = p^{O_1}_{x^3y^1} = 3,\; p^{O_1}_{x^2y^3} = p^{O_1}_{x^3y^2} = 1. \tag{A.6}$$

The 1st Chern numbers given in Table 4 can then be read directly from the values of fluxes given above. We now give similar data for the stacks $O_2, \ldots, O_8$:

$$F^{O_2}_{(1,1)} = -\frac{i}{2}\begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix}\begin{pmatrix} \frac{5}{2} & 4 & -3 \\ 4 & \frac{1}{2} & -1 \\ -3 & -1 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix}, \tag{A.7}$$

leading to:

$$|F^{U_5} + F^{O_2}| = 23\,, \quad |F^{U_5} - F^{O_2}| = -23\,, \quad |F^{O_2}| = \frac{195}{8}\,. \tag{A.8}$$

The oblique tadpoles are:

$$Q^{O_2}_{1\bar{2}} = 3 + 2\,, \quad Q^{O_2}_{2\bar{3}} = -12 + \frac{5}{2}\,, \quad Q^{O_2}_{3\bar{1}} = -4 + \frac{3}{2}\,, \tag{A.9}$$

while the diagonal tadpoles are:

$$Q^{O_2}_{1\bar{1}} = -\frac{5}{4}\,, \quad Q^{O_2}_{2\bar{2}} = -\frac{41}{4}\,, \quad Q^{O_2}_{3\bar{3}} = -\frac{59}{4}\,. \tag{A.10}$$

The fluxes in the real basis are:

$$p^{O_2}_{x^1y^1} = \frac{5}{2},\ p^{O_2}_{x^2y^2} = -p^{O_2}_{x^3y^3} = \frac{1}{2},\ p^{O_2}_{x^1y^2} = p^{O_2}_{x^2y^1} = 4,\ p^{O_2}_{x^1y^3} = p^{O_2}_{x^3y^1} = -3,\ p^{O_2}_{x^2y^3} = p^{O_2}_{x^3y^2} = -1. \tag{A.11}$$

$$F^{O_3}_{(1,1)} = -\frac{i}{2}\begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix}\begin{pmatrix} \frac{5}{2} & -4 & -3i \\ -4 & \frac{1}{2} & i \\ 3i & -i & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix}, \tag{A.12}$$

leading to

$$|F^{U_5} + F^{O3}| = 23\,, \quad |F^{U_5} - F^{O3}| = -23\,, \quad |F^{O3}| = \frac{195}{8}\,. \tag{A.13}$$

The oblique tadpoles are:

$$Q^{O_3}_{1\bar{2}} = -3 - 2\,, \quad Q^{O_3}_{2\bar{3}} = -12i + \frac{5i}{2}\,, \quad Q^{O_3}_{3\bar{1}} = -4i + \frac{3i}{2}\,, \tag{A.14}$$

and the diagonal ones are:

$$Q^{O_3}_{1\bar{1}} = -\frac{5}{4}\,, \quad Q^{O_3}_{2\bar{2}} = -\frac{41}{4}\,, \quad Q^{O_3}_{3\bar{3}} = -\frac{59}{4}\,. \tag{A.15}$$

The fluxes in the real basis are:

$$p^{O_3}_{x^1y^1} = \frac{5}{2},\ p^{O_3}_{x^2y^2} = -p^{O_3}_{x^3y^3} = \frac{1}{2},\ p^{O_3}_{x^1y^2} = p^{O_3}_{x^2y^1} = -4,\ p^{O_3}_{x^3x^1} = p^{O_3}_{y^3y^1} = 3,\ p^{O_3}_{x^2x^3} = p^{O_3}_{y^2y^3} = 1. \tag{A.16}$$

$$F^{O_4}_{(1,1)} = -\frac{i}{2}\begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix}\begin{pmatrix} \frac{5}{2} & -4 & 3i \\ -4 & \frac{1}{2} & -i \\ -3i & i & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix}, \tag{A.17}$$

leading to

$$|F^{U_5} + F^{O_4}| = 23\,, \quad |F^{U_5} - F^{O_4}| = -23\,, \quad |F^{O_4}| = \frac{195}{8}\,. \tag{A.18}$$

The oblique tadpoles are:

$$Q^{O_4}_{1\bar{2}} = -3 - 2\,, \quad Q^{O_4}_{2\bar{3}} = 12i - \frac{5i}{2}\,, \quad Q^{O_4}_{3\bar{1}} = 4i - \frac{3i}{2}\,, \tag{A.19}$$

and the diagonal tadpoles are:

$$Q^{O_4}_{1\bar{1}} = -\frac{5}{4}\,, \quad Q^{O_4}_{2\bar{2}} = -\frac{41}{4}\,, \quad Q^{O_4}_{3\bar{3}} = -\frac{59}{4}\,. \tag{A.20}$$

The fluxes in the real basis are:

$$p^{O_4}_{x^1y^1} = \frac{5}{2},\ p^{O_4}_{x^2y^2} = -p^{O_4}_{x^3y^3} = \frac{1}{2},\ p^{O_4}_{x^1y^2} = p^{O_4}_{x^2y^1} = -4,\ p^{O_4}_{x^3x^1} = p^{O_4}_{y^3y^1} = -3,\ p^{O_4}_{x^2x^3} = p^{O_4}_{y^2y^3} = -1. \tag{A.21}$$

The stacks $O_1, \ldots, O_4$, given above, satisfy the supersymmetry conditions (3.32). We now give the set of four stacks, $O_5, \ldots, O_8$, which satisfy the supersymmetry condition (3.38) for the values of J_i given in eqs. (3.36), (3.39):

$$F^{O_5}_{(1,1)} = -\frac{i}{2}\begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix}\begin{pmatrix} -\frac{25}{2} & -2i & -i \\ 2i & \frac{1}{2} & 1 \\ i & 1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix}; \tag{A.22}$$

$$|F^{U_5} + F^{O_5}| = 14\,, \quad |F^{U_5} - F^{O_5}| = -14\,, \quad |F^{O_5}| = \frac{87}{8}\,; \tag{A.23}$$

$$Q^{O_5}_{1\bar{2}} = i - i\,, \quad Q^{O_5}_{2\bar{3}} = 2 + \frac{25}{2}\,, \quad Q^{O_5}_{3\bar{1}} = -2i + \frac{i}{2}\,, \tag{A.24}$$

$$Q^{O_5}_{1\bar{1}} = -\frac{3}{4}\,, \quad Q^{O_5}_{2\bar{2}} = -\frac{29}{4}\,, \quad Q^{O_5}_{3\bar{3}} = -\frac{41}{4}\,; \tag{A.25}$$

$$p^{O_5}_{x^1y^1} = -\frac{25}{2},\ p^{O_5}_{x^2y^2} = p^{O_5}_{x^3y^3} = \frac{1}{2},\ p^{O_5}_{x^1x^2} = p^{O_5}_{y^1y^2} = -2,\ p^{O_5}_{x^3x^1} = p^{O_5}_{y^3y^1} = 1,\ p^{O_5}_{x^2y^3} = p^{O_5}_{x^3y^2} = 1. \tag{A.26}$$

$$F^{O_6}_{(1,1)} = -\frac{i}{2}\begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix}\begin{pmatrix} -\frac{25}{2} & -2i & i \\ 2i & \frac{1}{2} & -1 \\ -i & -1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix}; \tag{A.27}$$

$$|F^{U_5} + F^{O_6}| = 14\,, \quad |F^{U_5} - F^{O_6}| = -14\,, \quad |F^{O_6}| = \frac{87}{8}; \tag{A.28}$$

$$Q^{O_6}_{1\bar{2}} = i - i\,, \quad Q^{O_6}_{2\bar{3}} = -2 - \frac{25}{2}\,, \quad Q^{O_6}_{3\bar{1}} = 2i - \frac{i}{2}\,, \tag{A.29}$$

$$Q^{O_6}_{1\bar{1}} = -\frac{3}{4}\,, \quad Q^{O_6}_{2\bar{2}} = -\frac{29}{4}\,, \quad Q^{O_6}_{3\bar{3}} = -\frac{41}{4}\,; \tag{A.30}$$

$$p^{O_6}_{x^1y^1} = -\frac{25}{2},\ p^{O_6}_{x^2y^2} = p^{O_6}_{x^3y^3} = \frac{1}{2},\ p^{O_6}_{x^1x^2} = p^{O_6}_{y^1y^2} = -2,\ p^{O_6}_{x^3x^1} = p^{O_6}_{y^3y^1} = -1,\ p^{O_6}_{x^2y^3} = p^{O_6}_{x^3y^2} = -1. \tag{A.31}$$

$$F^{O_7}_{(1,1)} = -\frac{i}{2}\begin{pmatrix} dz_1 & dz_2 & dz_3 \end{pmatrix}\begin{pmatrix} -\frac{25}{2} & 2i & -1 \\ -2i & \frac{1}{2} & i \\ -1 & -i & \frac{1}{2} \end{pmatrix}\begin{pmatrix} d\bar{z}_1 \\ d\bar{z}_2 \\ d\bar{z}_3 \end{pmatrix}; \tag{A.32}$$

$$|F^{U_5}+F^{O_7}|=14\,,\quad |F^{U_5}-F^{O_7}|=-14\,,\quad |F^{O_7}|=\frac{87}{8}\,; \tag{A.33}$$

$$Q^{O_7}_{1\bar{2}}=-i+i\,,\quad Q^{O_7}_{2\bar{3}}=-2i-\frac{25i}{2}\,,\quad Q^{O_7}_{3\bar{1}}=-2+\frac{1}{2}\,, \tag{A.34}$$

$$Q^{O_7}_{1\bar{1}}=-\frac{3}{4}\,,\quad Q^{O_7}_{2\bar{2}}=-\frac{29}{4}\,,\quad Q^{O_7}_{3\bar{3}}=-\frac{41}{4}\,; \tag{A.35}$$

$$p^{O_7}_{x^1y^1}=-\frac{25}{2},\ p^{O_7}_{x^2y^2}=p^{O_7}_{x^3y^3}=\frac{1}{2},\ p^{O_7}_{x^1x^2}=p^{O_7}_{y^1y^2}=2,\ p^{O_7}_{x^1y^3}=p^{O_7}_{x^3y^1}=-1,\ p^{O_7}_{x^2x^3}=p^{O_7}_{y^2y^3}=1. \tag{A.36}$$

$$F^{O_8}_{(1,1)}=-\frac{i}{2}\begin{pmatrix}dz_1 & dz_2 & dz_3\end{pmatrix}\begin{pmatrix}-\frac{25}{2} & 2i & 1\\ -2i & \frac{1}{2} & -i\\ 1 & i & \frac{1}{2}\end{pmatrix}\begin{pmatrix}d\bar{z}_1\\ d\bar{z}_2\\ d\bar{z}_3\end{pmatrix}; \tag{A.37}$$

$$|F^{U_5}+F^{O_8}|=14\,,\quad |F^{U_5}-F^{O_8}|=-14\,,\quad |F^{O_8}|=\frac{87}{8}\,; \tag{A.38}$$

$$Q^{O_8}_{1\bar{2}}=-i+i\,,\quad Q^{O_8}_{2\bar{3}}=2i+\frac{25i}{2}\,,\quad Q^{O_8}_{3\bar{1}}=2-\frac{1}{2}\,, \tag{A.39}$$

$$Q^{O_8}_{1\bar{1}}=-\frac{3}{4}\,,\quad Q^{O_8}_{2\bar{2}}=-\frac{29}{4}\,,\quad Q^{O_8}_{3\bar{3}}=-\frac{41}{4}\,; \tag{A.40}$$

$$p^{O_8}_{x^1y^1}=-\frac{5}{2},\ p^{O_8}_{x^2y^2}=p^{O_8}_{x^3y^3}=\frac{1}{2},\ p^{O_8}_{x^1x^2}=p^{O8}_{y^1y^2}=2,\ p^{O_8}_{x^1y^3}=p^{O_8}_{x^3y^1}=1,\ p^{O_8}_{x^2x^3}=p^{O_8}_{y^2y^3}=-1. \tag{A.41}$$

B Complex structure moduli stabilization

For each stack of magnetized $D9$-branes, we have three complex conditions for the moduli of the complex structure derived from eq. (2.15).

From stack-O_1 :

$$4\tau_{11}+\frac{1}{2}\tau_{21}+\tau_{31} = \frac{5}{2}\tau_{12}+4\tau_{22}+3\tau_{32}, \tag{B.1}$$

$$3\tau_{11}+\tau_{21}-\frac{1}{2}\tau_{31} = \frac{5}{2}\tau_{13}+4\tau_{23}+3\tau_{33}, \tag{B.2}$$

$$3\tau_{12}+\tau_{22}-\frac{1}{2}\tau_{32} = 4\tau_{13}+\frac{1}{2}\tau_{23}+\tau_{33}. \tag{B.3}$$

From stack-O_2 :

$$
\begin{aligned}
4\tau_{11}+\frac{1}{2}\tau_{21}-\tau_{31} &= \frac{5}{2}\tau_{12}+4\tau_{22}-3\tau_{32}, && \text{(B.4)}\\
-3\tau_{11}-\tau_{21}-\frac{1}{2}\tau_{31} &= \frac{5}{2}\tau_{13}+4\tau_{23}-3\tau_{33}, && \text{(B.5)}\\
-3\tau_{12}-\tau_{22}-\frac{1}{2}\tau_{32} &= 4\tau_{13}+\frac{1}{2}\tau_{23}-\tau_{33}. && \text{(B.6)}
\end{aligned}
$$

From stack-O_3 :

$$
\begin{aligned}
-3\tau_{11}\tau_{32}+\tau_{21}\tau_{32}+3\tau_{31}\tau_{12}-\tau_{31}\tau_{22}+4\tau_{11}-\frac{1}{2}\tau_{21}+\frac{5}{2}\tau_{12}-4\tau_{22} &= 0, && \text{(B.7)}\\
-3\tau_{11}\tau_{33}+\tau_{21}\tau_{33}+3\tau_{13}\tau_{31}-\tau_{31}\tau_{23}+\frac{1}{2}\tau_{31}+\frac{5}{2}\tau_{13}-4\tau_{23}-3 &= 0, && \text{(B.8)}\\
-3\tau_{12}\tau_{33}+\tau_{22}\tau_{33}+3\tau_{13}\tau_{32}-\tau_{23}\tau_{32}+\frac{1}{2}\tau_{32}-4\tau_{13}+\frac{1}{2}\tau_{23}+1 &= 0. && \text{(B.9)}
\end{aligned}
$$

From stack-O_4 :

$$
\begin{aligned}
3\tau_{11}\tau_{32}-\tau_{21}\tau_{32}-3\tau_{31}\tau_{12}+\tau_{31}\tau_{22}+4\tau_{11}-\frac{1}{2}\tau_{21}+\frac{5}{2}\tau_{12}-4\tau_{22} &= 0, && \text{(B.10)}\\
3\tau_{11}\tau_{33}-\tau_{21}\tau_{33}-3\tau_{13}\tau_{31}+\tau_{31}\tau_{23}+\frac{1}{2}\tau_{31}+\frac{5}{2}\tau_{13}-4\tau_{23}+3 &= 0, && \text{(B.11)}\\
3\tau_{12}\tau_{33}-\tau_{22}\tau_{33}-3\tau_{13}\tau_{32}+\tau_{23}\tau_{32}+\frac{1}{2}\tau_{32}-4\tau_{13}+\frac{1}{2}\tau_{23}-1 &= 0. && \text{(B.12)}
\end{aligned}
$$

From stack-O_5 :

$$
\begin{aligned}
-2\tau_{11}\tau_{22}-\tau_{11}\tau_{32}+2\tau_{21}\tau_{12}+\tau_{31}\tau_{12}-\frac{1}{2}\tau_{21}-\tau_{31}-\frac{25}{2}\tau_{12}-2 &= 0, && \text{(B.13)}\\
-2\tau_{11}\tau_{23}-\tau_{11}\tau_{33}+2\tau_{21}\tau_{13}+\tau_{31}\tau_{13}-\tau_{21}-\frac{1}{2}\tau_{31}-\frac{25}{2}\tau_{13}-1 &= 0, && \text{(B.14)}\\
-2\tau_{12}\tau_{23}-\tau_{12}\tau_{33}+2\tau_{22}\tau_{13}+\tau_{32}\tau_{13}-\tau_{22}-\frac{1}{2}\tau_{32}+\frac{1}{2}\tau_{23}+\tau_{33} &= 0. && \text{(B.15)}
\end{aligned}
$$

From stack-O_6 :

$$
\begin{aligned}
-2\tau_{11}\tau_{22}+\tau_{11}\tau_{32}+2\tau_{21}\tau_{12}-\tau_{31}\tau_{12}-\frac{1}{2}\tau_{21}+\tau_{31}-\frac{25}{2}\tau_{12}-2 &= 0, && \text{(B.16)}\\
-2\tau_{11}\tau_{23}+\tau_{11}\tau_{33}+2\tau_{21}\tau_{13}-\tau_{31}\tau_{13}+\tau_{21}-\frac{1}{2}\tau_{31}-\frac{25}{2}\tau_{13}+1 &= 0, && \text{(B.17)}\\
-2\tau_{12}\tau_{23}+\tau_{12}\tau_{33}+2\tau_{22}\tau_{13}-\tau_{32}\tau_{13}+\tau_{22}-\frac{1}{2}\tau_{32}+\frac{1}{2}\tau_{23}-\tau_{33} &= 0. && \text{(B.18)}
\end{aligned}
$$

From stack-O_7 :

$$
\begin{aligned}
2\tau_{11}\tau_{22} - 2\tau_{21}\tau_{12} + \tau_{21}\tau_{32} - \tau_{22}\tau_{31} - \frac{1}{2}\tau_{21} - \frac{25}{2}\tau_{12} - \tau_{32} + 2 &= 0, && \text{(B.19)}\\
2\tau_{11}\tau_{23} - 2\tau_{21}\tau_{13} + \tau_{21}\tau_{33} - \tau_{23}\tau_{31} + \tau_{11} - \frac{1}{2}\tau_{31} - \frac{25}{2}\tau_{13} - \tau_{33} &= 0, && \text{(B.20)}\\
2\tau_{12}\tau_{23} - 2\tau_{22}\tau_{13} + \tau_{22}\tau_{33} - \tau_{23}\tau_{32} + \tau_{12} - \frac{1}{2}\tau_{32} + \frac{1}{2}\tau_{23} + 1 &= 0. && \text{(B.21)}
\end{aligned}
$$

From stack-O_8 :

$$
\begin{aligned}
2\tau_{11}\tau_{22} - 2\tau_{21}\tau_{12} - \tau_{21}\tau_{32} + \tau_{22}\tau_{31} - \frac{1}{2}\tau_{21} - \frac{25}{2}\tau_{12} + \tau_{32} + 2 &= 0, && \text{(B.22)}\\
2\tau_{11}\tau_{23} - 2\tau_{21}\tau_{13} - \tau_{21}\tau_{33} + \tau_{23}\tau_{31} - \tau_{11} - \frac{1}{2}\tau_{31} - \frac{25}{2}\tau_{13} + \tau_{33} &= 0, && \text{(B.23)}\\
2\tau_{12}\tau_{23} - 2\tau_{22}\tau_{13} - \tau_{22}\tau_{33} + \tau_{23}\tau_{32} - \tau_{12} - \frac{1}{2}\tau_{32} + \frac{1}{2}\tau_{23} - 1 &= 0. && \text{(B.24)}
\end{aligned}
$$

Now, from stack-O_1 and stack-O_2 one obtains from eqs. (B.1) and (B.4):

$$\tau_{31} = 3\tau_{32}\,, \tag{B.25}$$

and

$$4\tau_{11} + \frac{1}{2}\tau_{21} = \frac{5}{2}\tau_{12} + 4\tau_{22}\,; \tag{B.26}$$

from eqs. (B.2) and (B.5):

$$3\tau_{11} + \tau_{21} = 3\tau_{33}\,, \tag{B.27}$$

and

$$-\frac{1}{2}\tau_{31} = \frac{5}{2}\tau_{13} + 4\tau_{23}\,; \tag{B.28}$$

and from eqs. (B.3) and (B.6):

$$3\tau_{12} + \tau_{22} = \tau_{33}\,, \tag{B.29}$$

and

$$-\frac{1}{2}\tau_{32} = 4\tau_{13} + \frac{1}{2}\tau_{23}\,; \tag{B.30}$$

Similarly, from stack-O_3 and stack-O_4 one has, by adding eqs. (B.7) and (B.10):

$$4\tau_{11} - \frac{1}{2}\tau_{21} + \frac{5}{2}\tau_{12} - 4\tau_{22} = 0\,; \tag{B.31}$$

by adding eqs. (B.8) and (B.11):

$$\frac{1}{2}\tau_{31} + \frac{5}{2}\tau_{13} - 4\tau_{23} = 0\,; \tag{B.32}$$

and by adding eqs. (B.9) and (B.12):

$$\frac{1}{2}\tau_{32} - 4\tau_{13} + \frac{1}{2}\tau_{23} = 0. \tag{B.33}$$

Use of eqs. (B.30) and (B.33) gives:

$$\tau_{13} = 0\,, \tag{B.34}$$

and

$$\tau_{32} + \tau_{23} = 0\,. \tag{B.35}$$

Moreover, one has from eqs. (B.34) and (B.32):

$$\tau_{31} = 8\tau_{23}\,; \tag{B.36}$$

from eqs. (B.36) and (B.25):

$$3\tau_{32} = 8\tau_{23}\,; \tag{B.37}$$

from eqs. (B.37) and (B.35):

$$\tau_{32} = \tau_{23} = 0\,; \tag{B.38}$$

and from eqs. (B.38) and (B.36):

$$\tau_{31} = 0\,. \tag{B.39}$$

Similarly, use of eqs. (B.26) and (B.31) implies:

$$\tau_{21} = 5\tau_{12}\,, \tag{B.40}$$

and

$$\tau_{11} = \tau_{22}\,; \tag{B.41}$$

while use of eq. (B.41) in eqs. (B.27) and (B.29) gives:

$$3\tau_{11} + \tau_{21} - 3\tau_{33} = 0\,, \tag{B.42}$$

and

$$3\tau_{11} + 9\tau_{12} - 3\tau_{33} = 0\,. \tag{B.43}$$

Eqs. (B.42) and (B.43) give:

$$\tau_{21} = 9\tau_{12}\,, \tag{B.44}$$

which comparing with eq. (B.40) implies:

$$\tau_{21} = \tau_{12} = 0\,. \tag{B.45}$$

Using the result of eq. (B.45) into eq. (B.42) then gives (using also eq. (B.41)),

$$\tau_{11} = \tau_{22} = \tau_{33} \equiv \tau\,. \tag{B.46}$$

The value of τ is finally determined from any of the bilinear equations, such as eq. (B.8) or (B.9):

$$\tau = i\,. \tag{B.47}$$

C Kähler class moduli stabilization

For the stabilization of Kähler class, let us denote for definiteness the volume of the 4-cycles associated to $J \wedge J$ as

$$(J \wedge J)_{i\bar{j}} = V_{i\bar{j}}\,, \tag{C.1}$$

where the indices $i, \bar{j}$ correspond to the $(1,1)$-cycle perpendicular to the given 4-cycle. In the above notation, the supersymmetry conditions on the Kähler moduli for the various stacks read as follows :

From stack-O_1 using eq. (A.2):

$$\frac{195}{8} - \left[\frac{5}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} - \frac{1}{2}V_{3\bar{3}} + 4V_{1\bar{2}} + 4V_{2\bar{1}} + 3V_{1\bar{3}} + 3V_{3\bar{1}} + V_{2\bar{3}} + V_{3\bar{2}}\right] = 0, \tag{C.2}$$

from stack-O_2 using eq. (A.7):

$$\frac{195}{8} - \left[\frac{5}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} - \frac{1}{2}V_{3\bar{3}} + 4V_{1\bar{2}} + 4V_{2\bar{1}} - 3V_{1\bar{3}} - 3V_{3\bar{1}} - V_{2\bar{3}} - V_{3\bar{2}}\right] = 0, \tag{C.3}$$

from stack-O_3 using eq. (A.12):

$$\frac{195}{8} - \left[\frac{5}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} - \frac{1}{2}V_{3\bar{3}} - 4V_{1\bar{2}} - 4V_{2\bar{1}} - 3iV_{1\bar{3}} + 3iV_{3\bar{1}} + iV_{2\bar{3}} - iV_{3\bar{2}}\right] = 0, \tag{C.4}$$

from stack-O_4 using eq. (A.17):

$$\frac{195}{8} - \left[\frac{5}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} - \frac{1}{2}V_{3\bar{3}} - 4V_{1\bar{2}} - 4V_{2\bar{1}} + 3iV_{1\bar{3}} - 3iV_{3\bar{1}} - iV_{2\bar{3}} + iV_{3\bar{2}}\right] = 0, \qquad (C.5)$$

from stack-O_5 using eq. (A.22):

$$\frac{87}{8} - \left[\frac{-25}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} + \frac{1}{2}V_{3\bar{3}} - 2iV_{1\bar{2}} + 2iV_{2\bar{1}} - iV_{1\bar{3}} + iV_{3\bar{1}} + V_{2\bar{3}} + V_{3\bar{2}}\right] = 0, \qquad (C.6)$$

from stack-O_6 using eq. (A.27):

$$\frac{87}{8} - \left[\frac{-25}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} + \frac{1}{2}V_{3\bar{3}} - 2iV_{1\bar{2}} + 2iV_{2\bar{1}} + iV_{1\bar{3}} - iV_{3\bar{1}} - V_{2\bar{3}} - V_{3\bar{2}}\right] = 0, \qquad (C.7)$$

from stack-O_7 using eq. (A.32):

$$\frac{87}{8} - \left[\frac{-25}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} + \frac{1}{2}V_{3\bar{3}} + 2iV_{1\bar{2}} - 2iV_{2\bar{1}} - V_{1\bar{3}} - V_{3\bar{1}} + iV_{2\bar{3}} - iV_{3\bar{2}}\right] = 0, \qquad (C.8)$$

from stack-O_8 using eq. (A.37):

$$\frac{87}{8} - \left[\frac{-25}{2}V_{1\bar{1}} + \frac{1}{2}V_{2\bar{2}} + \frac{1}{2}V_{3\bar{3}} + 2iV_{1\bar{2}} - 2iV_{2\bar{1}} + V_{1\bar{3}} + V_{3\bar{1}} - iV_{2\bar{3}} + iV_{3\bar{2}}\right] = 0. \qquad (C.9)$$

Now, from stacks-O_1 and O_2, eqs. (C.2) and (C.3) give:

$$3\left(V_{1\bar{3}} + V_{3\bar{1}}\right) + \left(V_{2\bar{3}} + V_{3\bar{2}}\right) = 0; \qquad (C.10)$$

from stacks-O_3 and O_4, eqs. (C.4) and (C.5) give:

$$-3i\left(V_{1\bar{3}} - V_{3\bar{1}}\right) + i\left(V_{2\bar{3}} - V_{3\bar{2}}\right) = 0; \qquad (C.11)$$

from stacks-O_5 and O_6, eqs. (C.6) and (C.7) give:

$$-i\left(V_{1\bar{3}} - V_{3\bar{1}}\right) + \left(V_{2\bar{3}} + V_{3\bar{2}}\right) = 0; \qquad (C.12)$$

and from stacks-O_7 and O_8, eqs. (C.8) and (C.9) give:

$$-\left(V_{1\bar{3}} + V_{3\bar{1}}\right) + i\left(V_{2\bar{3}} - V_{3\bar{2}}\right) = 0. \qquad (C.13)$$

Eq. (C.13) implies

$$i\left(V_{2\bar{3}} - V_{3\bar{2}}\right) = \left(V_{1\bar{3}} + V_{3\bar{1}}\right), \qquad (C.14)$$

which leads from eq. (C.10)

$$3i\,(V_{2\bar{3}} - V_{3\bar{2}}) + (V_{2\bar{3}} + V_{3\bar{2}}) = 0\,. \tag{C.15}$$

Similarly, eq.(C.12) implies

$$i\,(V_{1\bar{3}} - V_{3\bar{1}}) = (V_{2\bar{3}} + V_{3\bar{2}})\ , \tag{C.16}$$

which leads from eq. (C.11)

$$-\,3\,(V_{2\bar{3}} + V_{3\bar{2}}) + i\,(V_{2\bar{3}} - V_{3\bar{2}}) = 0\,. \tag{C.17}$$

Now eqs. (C.15) and (C.17) can be solved to give

$$V_{2\bar{3}} + V_{3\bar{2}} = 0, \tag{C.18}$$

and

$$V_{2\bar{3}} - V_{3\bar{2}} = 0, \tag{C.19}$$

implying

$$V_{2\bar{3}} = V_{3\bar{2}} = 0. \tag{C.20}$$

Then one has from eq. (C.10)

$$V_{1\bar{3}} + V_{3\bar{1}} = 0, \tag{C.21}$$

and from eq. (C.11)

$$V_{1\bar{3}} - V_{3\bar{1}} = 0, \tag{C.22}$$

implying

$$V_{1\bar{3}} = V_{3\bar{1}} = 0. \tag{C.23}$$

Using the obtained values, eqs. (C.2) - (C.4) give

$$V_{1\bar{2}} + V_{2\bar{1}} = 0, \tag{C.24}$$

while eqs. (C.8) - eq. (C.6) give

$$V_{1\bar{2}} - V_{2\bar{1}} = 0, \tag{C.25}$$

implying

$$V_{1\bar{2}} = V_{2\bar{1}} = 0. \tag{C.26}$$

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